

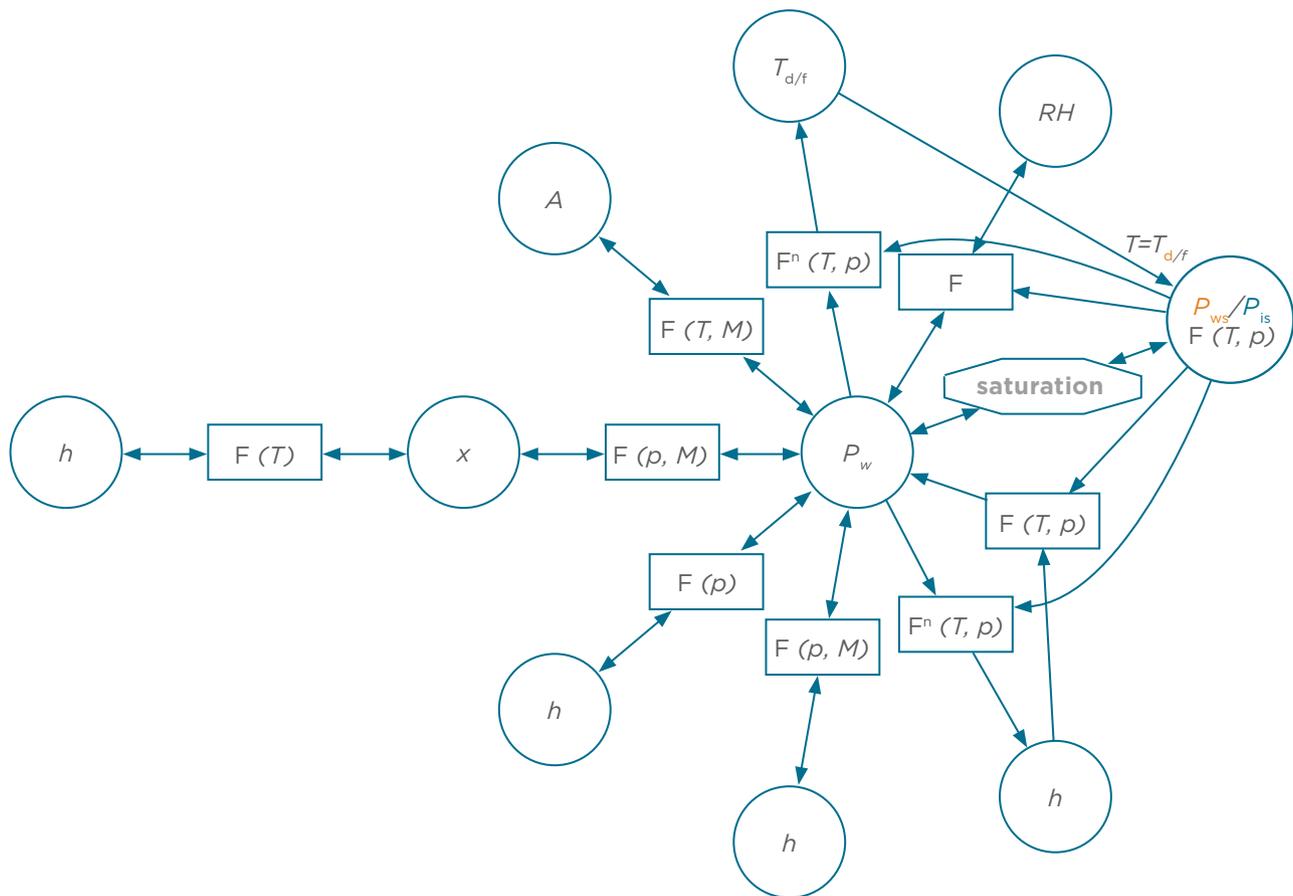
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Humidity conversions

1. Foreword

This document describes the functions and methods for calculating different humidity variables. As shown in the diagram below, the partial vapor pressure (P_w) and saturation vapor pressure (P_{ws}/P_{is}) are the key factors in parametric calculations. Other parameters can be derived from these parameters by using functions, iterative methods, and the auxiliary parameters: temperature and pressure. This document uses units from the International System of Units (SI); conversions to imperial units can be found in chapter 11.



- P_{ws}/P_{is} = Vapor saturation pressure over water / ice in Pa
- P_w = Vapor pressure in Pa
- RH = Relative humidity
- $T_{d/f}$ = Dew point / Frost point in °C
- A = Absolute humidity in g/m³
- x = Mixing ratio in g/kg
- h = Enthalpy in kJ/kg
- ppm_v = Parts per million (volume H_2O /volume gas)
- ppm_m = Parts per million (mass H_2O /mass gas)
- $T_{w/i}$ = Wet bulb / Frost bulb temperature in °C
- $F(T, p, M)$ = Function with auxiliary quantities: temperature, pressure, or molar mass
- $F^n(T, p)$ = Iterative method with auxiliary quantities: temperature and pressure

2. Water vapor saturation pressure - P_{ws} and P_{is}

Many saturation vapor pressure equations exist to calculate the saturation pressure of water vapor over a surface of liquid water or ice. Water vapor saturation pressure can be calculated using, for example Sonntag's equation [1]. This equation is valid in temperature range from -100 °C to 100 °C.

$$\left. \begin{matrix} e_{ws}(T_K) \\ e_{is}(T_K) \end{matrix} \right\} = a_0 \cdot \exp \left[\frac{a_1}{T_K} + a_2 + a_3 T_K + a_4 T_K^2 + a_5 \ln(a_6 \cdot T_K) \right], \text{ where} \quad (1)$$

e_{ws} = Saturation vapor pressure over water in Pa

e_{is} = Saturation vapor pressure above ice in Pa

T_K = Temperature in K

p = Total pressure in Pa

Table 1. Numerical values for coefficients $a_0 \dots a_6$

Coefficients a_n	Over water (P_{ws}) (0 to 100 °C)	Over ice (P_{is}) (-100 to 0 °C)
a_0	1 Pa	1 Pa
a_1	-6.0969385E+03 K	-6.0245282E+03 K
a_2	2.12409642E+01	2.932707E+01
a_3	-2.711193E-02 K ⁻¹	1.0613868E-02 K ⁻¹
a_4	1.673952E-05 K ⁻²	-1.3198825E-05 K ⁻²
a_5	2.433502	-4.9382577E-01
a_6	1 K ⁻¹	1 K ⁻¹

In this document the equation is supplemented with an enhancement factor (f) to cover a pressure range from 0 to 20 MPa.

$$\begin{aligned} P_{ws}(T_K, p) &= e_{ws} \cdot f(T_K - 273.15, p), \text{ or} \\ P_{is}(T_K, p) &= e_{is} \cdot f(T_K - 273.15, p), \text{ where} \end{aligned} \quad (2)$$

P_{ws} = Saturation vapor pressure above water in Pa

P_{is} = Saturation vapor pressure above ice in Pa

f = Enhancement factor, see next chapter.

Example:

What is the vapor saturation pressure at 20 °C?

$T = 20 \text{ °C} \Rightarrow T_K = 20 + 273.15 = 293.15 \text{ K}$

$p = \text{Sea level} = 101325 \text{ Pa}$

$P_{ws} = 2339.25 \text{ Pa}$

2.1 Enhancement factor - f

Water vapor pressure describes a situation where at least two components form a gas mixture. Thus, the gas cannot be considered an ideal gas and an enhancement factor (f) is introduced to correct the non-ideality effects of humid gases.

For atmospheric pressure and vacuum pressures this effect is typically negligible, but as the gas pressure increases so do the non-ideality effects. For air pressures greater than 1100 hPa, the formula developed by Lewis Greenspan [2] is typically used to determine the enhancement factor. Coefficients used in this document are calculated by Bob Hardy [3]. These coefficients can also be used to calculate the enhancement factor in nitrogen if slightly higher uncertainty is acceptable.

$$f(T, P) = \exp \left[\alpha(T) \cdot \left(1 - \frac{e_{ws/is}}{P} \right) + \beta(T) \cdot \left(\frac{P}{e_{ws/is}} - 1 \right) \right] \quad (3)$$

$$\alpha(T) = \sum_{i=0}^3 a_i \cdot T^i \quad (3.1)$$

$$\beta(T) = \exp \left[\sum_{i=0}^3 b_i \cdot T^i \right], \text{ where} \quad (3.2)$$

- e_{ws} = Saturation vapor pressure over water in Pa
- e_{is} = Saturation vapor pressure over ice in Pa
- T = Temperature in °C
- ρ = Total pressure in Pa

Table 2. Numerical values for coefficients a0 ... a3 and b0 ... b3 [3]

Coefficients a_n & b_n	Over water (0 to 100 °C)	Over supercooled water (-50 to 0 °C)	Over ice (-100 to 0 °C)
a_0	3.5362400E-04	3.6218300E-4	3.6444900E-04
a_1	2.9328363E-05 °C ⁻¹	2.6061244E-5 °C ⁻¹	2.9367585E-05 °C ⁻¹
a_2	2.6168979E-07 °C ⁻²	3.8667770E-7 °C ⁻²	4.8874766E-07 °C ⁻²
a_3	8.5813609E-09 °C ⁻³	3.8268958E-9 °C ⁻³	4.3669918E-09 °C ⁻³
b_0	-1.0758800E+01	-1.0760400E+1	-1.0727100E+01
b_1	6.3268134E-02 °C ⁻¹	6.3987441E-2 °C ⁻¹	7.6215115E-02 °C ⁻¹
b_2	-2.5368934E-04 °C ⁻²	-2.6351566E-4 °C ⁻²	-1.7490155E-04 °C ⁻²
b_3	6.3405286E-07 °C ⁻³	1.6725084E-6 °C ⁻³	2.4668279E-06 °C ⁻³

Example:

$T = 20$ °C

$\rho = 10$ bar(g) = 1 MPa = 1 000 000 Pa

$f = 1.0307$

3. Relative humidity – RH

Relative humidity (RH) is defined at all temperatures and pressures as the ratio of the water vapor pressure (P_w) to the saturation water vapor pressure over water (P_{ws}) at the gas temperature:

$$RH = \frac{P_w}{P_{ws}} \cdot 100 \%rh \quad (4)$$

The total pressure is not part of the definition. Above 100 °C the same definition is valid. But as the vapor saturation pressure P_{ws} is greater than 1 013 hPa (normal ambient pressure), the RH can't reach 100% in an unpressurized system. Below 0 °C the definition is also valid. Here 100%rh is also impossible because condensation will occur at a humidity level lower than 100% (when the vapor is saturated against ice). Relative humidity over ice (RH_i) can be calculated by using coefficients for P_{is} :

$$RH_i = \frac{P_w}{P_{is}} \cdot 100 \%rh_i \quad (5)$$

4. Dew point and frost point – $T_{d/f}$

The dew point temperature (T_d) is the temperature to which air must be cooled to become saturated with water vapor. The frost point (T_f) is the temperature at which the air is saturated by water vapor over an ice surface. The term $T_{d/f}$ means that when the dew point temperature is equal to or above 0 °C the dew point calculation (T_d) is used, with the (T_f) calculation used below 0 °C.

4.1 Dew point to vapor pressure – T_d to P_w

Calculating the vapor pressure from T_d is a straightforward operation. Use equation (1) to calculate the vapor saturation pressure in the T_d condition:

$$P_w(T_K) = P_{ws}((T_d + 273.15)) \quad (6)$$

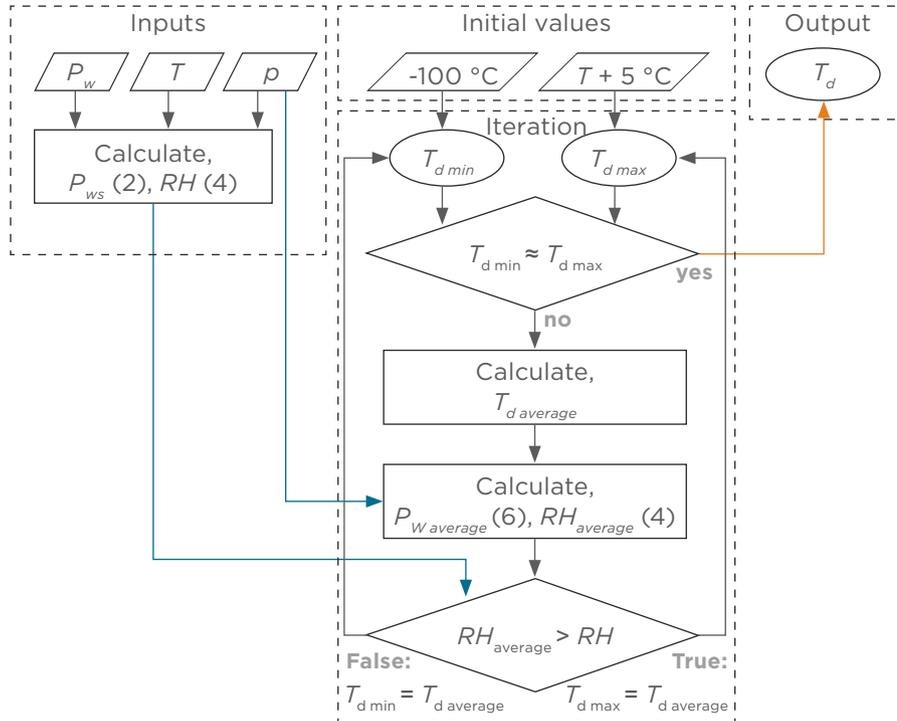
4.2 Vapor pressure to dew point – P_w to T_d

Calculating dew point from vapor pressure, temperature, and pressure involves complex equations, because both the enhancement factor and the vapor pressure will be dependent on the dew point itself. Dew point can be calculated using an iterative procedure:

Initial values for the trial-and-error process should be wide enough to accommodate the result within its boundaries. Proper values for majority of applications are as follows.

$$T_{d \min} = -100 \text{ } ^\circ\text{C}, T_{d \max} = T + 5 \text{ } ^\circ\text{C}, \text{ but at a maximum of } 100 \text{ } ^\circ\text{C}$$

In this document $T_{d,min}$ is limited to $-100\text{ }^{\circ}\text{C}$, because this is the limit of specified accuracy for the P_{ws} calculation (1). Here, a small margin ($+5\text{ }^{\circ}\text{C}$) is introduced in case there are inaccuracies in the initial parameters. The vapor pressure to dew point iterative process is shown in the flow diagram below:



Example:

Calculate T_d in the following conditions:

$$\begin{aligned}
 P_w &= 233.92 \text{ Pa} \\
 T &= 20\text{ }^{\circ}\text{C} \Rightarrow T_k = 293.15 \text{ K} \\
 p &= 101325 \text{ Pa}
 \end{aligned}$$

$$\begin{aligned}
 P_{ws} (1) &= 2339.25 \text{ Pa} \\
 RH (3) &= 10.0 \text{ \%rh}
 \end{aligned}$$

Iteration					
Step	T_d min °C	T_d max °C	Average		
			T_d °C	P_w Pa	RH %rh
1	-100 [initial]	25 [T+5 initial]	-37.5	24.68	1.051
2	-37.5	25	-6.25	385.0	16.39
3	-37.5	-6.25	-21.9	107.2	4.563
4	-21.9	-6.25	-14.1	207.5	8.837
5	-14.1	-6.25	-10.2	284.1	12.10
6	-14.1	-10.2	-12.1	243.1	10.35
7	-14.1	-12.1	-13.1	224.7	9.568
8	-13.1	-12.1	-12.6	233.7	9.954
..
19	-12.5404	-12.5400	-12.5402	234.9	10.00
20	-12.5404	-12.5402	-12.5403	234.9	10.00

Answer: $T_d = -12.540$ °C

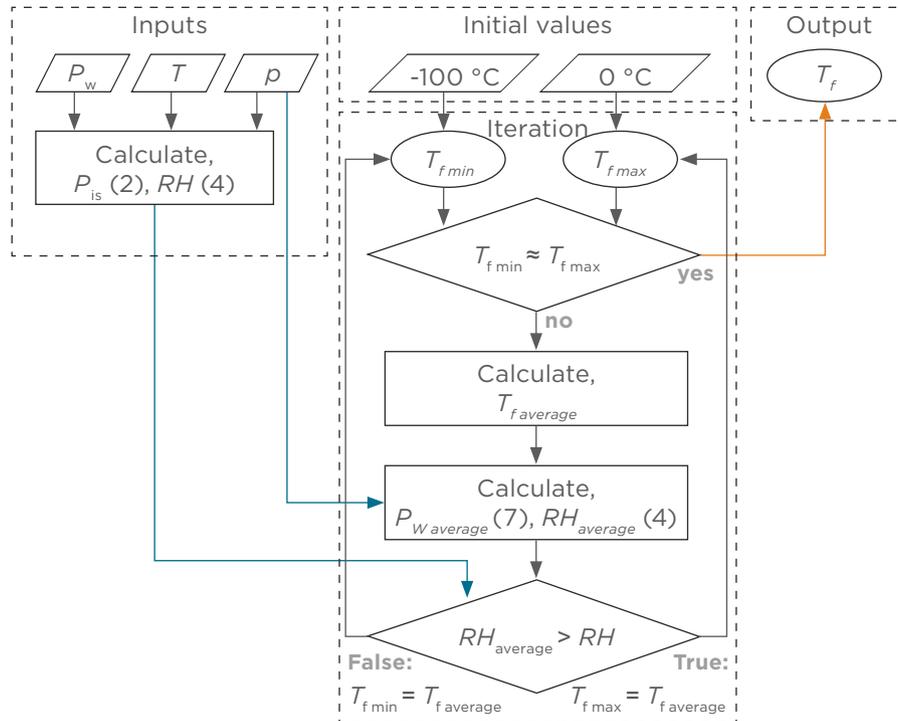
4.3 Frost point to vapor pressure - T_f to P_w

Use “over ice” coefficients (1) for converting frost point into water vapor pressure:

$$P_w(T_K) = P_{is} \left((T_f + 273.15) \right) \quad (7)$$

4.4 Vapor pressure to frost point - P_w to T_f

Frost point (T_f) is equal to dew point (T_d), if $T_d \geq 0$ °C. In other cases, T_f can be calculated using a similar method to T_d , but using coefficients over ice, P_{is} (1):



Example:

In the example calculation in chapter 4.2, the T_d result was below zero ($T_d = -12.540$ °C). Calculate T_f using the same initial values:

$$\begin{aligned}
 P_w &= 233.92 \text{ Pa} \\
 T &= 20 \text{ °C} \Rightarrow T_k = 293.15 \text{ K} \\
 p &= 101325 \text{ Pa} \\
 P_{is} (1) &= 2339.25 \text{ Pa} \\
 RH (3) &= 10.0 \text{ \%rh}
 \end{aligned}$$

Answer: $T_f = -11.18$ °C

5. Absolute humidity - A

Absolute humidity is defined as the mass of water vapor in a certain volume. If ideal gas behavior is assumed, then absolute humidity can be calculated using:

$$A = \frac{M_{H_2O} \cdot P_w}{R \cdot T_K}, \text{ where} \quad (8)$$

- A = Absolute humidity in g/m^3
- M_{H_2O} = Molecular mass of water in g/mol
- R = Universal gas constant $8.3145 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$
- P_w = Vapor pressure in Pa
- T_K = Temperature in K

Example:

Calculate the absolute humidity of air in the following conditions:

- RH = 80.0%rh
- T = 20 °C $\Rightarrow T_k = 293.15 \text{ K}$
- P = 101325 Pa

Molar mass of water = 18.0154 g/mol ¹

- $P_{ws}(1)$ = 2339.25 Pa
- $P_w(5)$ = 1871.51 Pa

$$A = \frac{18.0154 \frac{\text{g}}{\text{mol}} \cdot 1871.51 \text{ Pa}}{8.3145 \text{ J K}^{-1} \text{ mol}^{-1} \cdot 293.15 \text{ K}} = 13.8 \text{ g/m}^3$$

6. Mixing ratio - x

The mixing ratio (mass of water vapor/mass of dry gas) is calculated using:

$$x = B \frac{P_w}{P - P_w}, \quad (9)$$

x = Mixing ratio $\text{g}_{H_2O}/\text{kg}_{\text{dry gas}}$

$$B = \frac{M_{H_2O}}{M_{gas}} \cdot 1000 \text{ in g/kg, where}$$

- M_{H_2O} = Molecular weight of water = 18.0154 g/mol ¹
- M_{gas} = Molecular weight of gas in g/mol (air = 28.965 g/mol ¹ -> Bair = 621.97 g/kg)
- P_w = Vapor pressure in Pa
- P = Total pressure in Pa

¹See chapter 10 Molecular weight

B can be calculated for other gases by using their molecular weight. An example calculation for hydrogen (H_2) is [4]:

$$B = \frac{M_{H_2O}}{M_{H_2}} \cdot 1000 = \frac{18.0154 \text{ g/mol}}{2.0156 \text{ g/mol}} \cdot 1000 = 8\,938.0 \text{ g/kg}$$

Example:

Calculate mixing ratio in the following conditions:

$$\begin{aligned} RH &= 50.0\%rh \\ T &= 80 \text{ }^\circ\text{C} \Rightarrow T_k = 353.15 \text{ K} \\ p &= 101325 \text{ Pa} \end{aligned}$$

$$\begin{aligned} P_{ws}(1) &= 47415.54 \text{ Pa} \\ P_w(5) &= 23703.69 \text{ Pa} \end{aligned}$$

$$x = 621.97 \cdot 23703.69 / (101325 - 23703.69) = 189.93 \text{ g/kg}$$

7. Enthalpy - h

Specific enthalpy of water vapor in a constant pressure can be calculated from mixing ratio using a following formula. Note that the coefficients given in the following equation are depended on temperature and pressure. These constants will give accurate results in ambient pressure. hwe Accuracy can be improved by using a temperature specific value from a steam table:

$$h = C_{pg} \cdot T + \frac{x}{1000} (C_{pw} \cdot T + h_{we}), \text{ where} \quad (10)$$

$$\begin{aligned} h &= \text{Enthalpy in kJ/kg} \\ C_{pg} &= \text{Specific heat capacity of gas at constant pressure: air} = 1.006 \text{ kJ/kg}^\circ\text{C} \\ C_{pw} &= \text{Specific heat capacity of water vapor at constant pressure at } 0 \text{ }^\circ\text{C temperature: } 1.84 \text{ kJ/kg}^\circ\text{C} \\ h_{we} &= \text{Evaporation heat of water } 0 \text{ }^\circ\text{C: } 2501 \text{ kJ/kg} \\ x &= \text{Mixing ratio } \frac{\text{g}_{H_2O}}{\text{kg}_{dry\ gas}} T \\ T &= \text{Temperature in } ^\circ\text{C} \end{aligned}$$

Example:

Calculate enthalpy in the following conditions:

$$x = 190 \text{ g/kg}$$

$$T = 80 \text{ }^\circ\text{C}$$

$$h = C_{pg} \cdot T + \frac{x}{1000} (C_{pw} + h_{we} \cdot T),$$

8. H₂O content – ppm (parts per million)

There are four different ways to express H₂O content and it is important to use the correct one. H₂O content may refer to either mass fraction or volume fraction. In addition to this, ppm can be expressed as a proportion of dry gas or as a proportion of total gas. These four different variations are described in the following chapters.

8.1 ppm_{V dry} – Volume_{H₂O}/volume_{dry}

$$ppm_{V \text{ dry}} = \frac{P_w}{p - P_w} \cdot 10^6, \text{ where} \quad (11)$$

P_w = Vapor pressure in Pa

p = Total pressure in Pa

8.2 ppm_{m dry} – Mass_{H₂O}/mass_{dry}

$$ppm_{m \text{ dry}} = \frac{M_{H_2O} \cdot P_w}{M_{gas} \cdot (p - P_w)} \cdot 10^6, \text{ where} \quad (12)$$

M_{H_2O} = Molecular weight of water in g/mol (18.0146)

M_{gas} = Molecular weight of dry gas in g/mol (air = 28.965)²

8.3 ppm_{V wet} – Volume_{H₂O}/volume_{wet}

$$ppm_{V \text{ wet}} = \frac{P_w}{p} \cdot 10^6 \quad (13)$$

8.4 ppm_{m wet} – Mass_{H₂O}/mass_{wet}

$$ppm_{m \text{ wet}} = \frac{M_{H_2O} \cdot P_w}{M_{gas} \cdot p} \cdot 10^6 \quad (14)$$

²See chapter 10 Molecular weight

9. Wet bulb temperature - T_w

The cooling effect of a wet surface can be used to measure humidity with a device called a psychrometer. Nowadays these devices have mainly been replaced by modern capacitive thin film hygrometers. However, this parameter is utilized in other application areas that use evaporative cooling. Specifying the accuracy for this parameter is challenging as there are many uncertainties in vaporization, such as ventilation, materials, etc., but together these can be taken into account to determine a psychrometer coefficient. This document will use a coefficient that has been well established in Vaisala humidity instruments.

9.1 Wet bulb to vapor pressure - T_w to P_w

The dry bulb temperature T_{dry} and the wet bulb temperature T_{wet} can be converted into P_w

using the following formula:

$$P_w = P_{ws}(T_w, p) - p \cdot K \cdot (T - T_w), \text{ where} \quad (15)$$

P_w = Vapor pressure in Pa
 T_w = Wet bulb in °C
 p = Total pressure in Pa
 K = Psychrometer coefficient for water: 0.000662 °C⁻¹

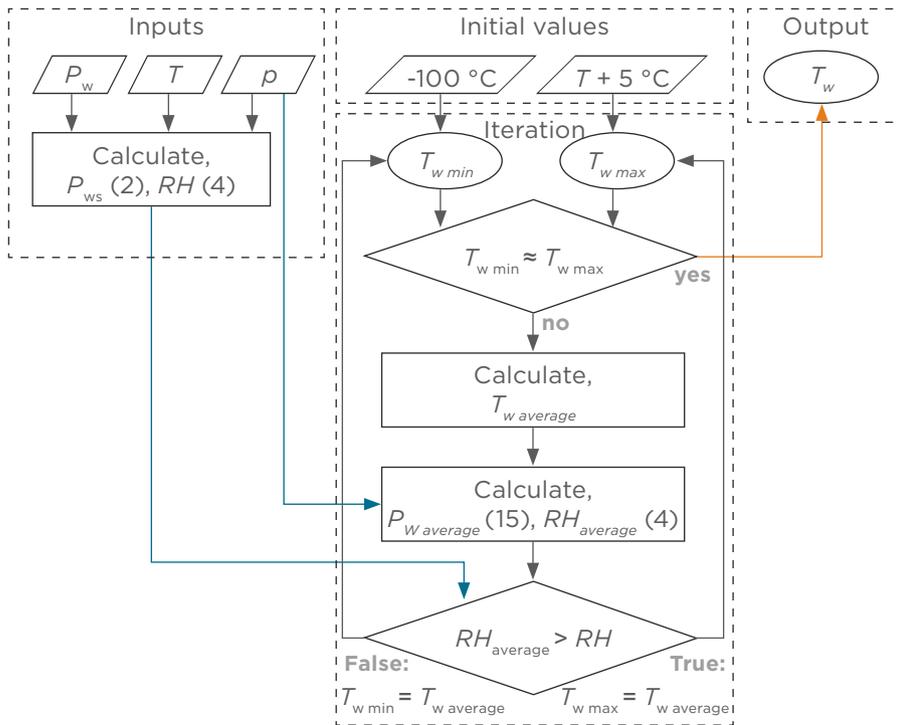
9.2 Vapor pressure to wet bulb - P_w to T_w

Calculating wet bulb from vapor pressure, temperature, and pressure involves complex equations, because both the enhancement factor and the vapor pressure will be dependent on the wet bulb result. Wet bulb can be calculated using an iterative procedure:

Initial values for the trial-and-error process should be wide enough to accommodate the result within its boundaries. Proper values for the majority of applications are as follows.

$$T_{d \min} = -100 \text{ °C}, T_{d \max} = T + 5 \text{ °C}, \text{ but at the maximum of } 100 \text{ °C}$$

In this document $T_{d \min}$ is limited to -100 °C because this is the limit of specified accuracy for the P_{ws} calculation (1). In practice, a wet bulb reading that is below 0 °C shouldn't be used and a frost bulb calculation should be used instead. Here a small margin (+5 °C) is introduced in case there are inaccuracies in the initial parameters. The iterative process of vapor pressure to wet bulb is shown in the following flow diagram:



Example:

Calculate wet bulb in the following conditions:

$$RH = 10.0\ \%rh$$

$$T = 20\text{ °C} \Rightarrow Tk = 293.15\text{ K}$$

$$P = 101325\text{ Pa}$$

$$P_{ws}(1) = 2339.25\text{ Pa}$$

$$\text{Answer: } T_w = 7.8\text{ °C}$$

10. Molecular weight

The molecular weight of a substance (molar mass) M is needed in mass-based humidity calculations such as absolute humidity and mixing ratio. The molecular mass of a substance is the sum of the molar fractions of gas components multiplied by their molar mass [4]. Let's take dry air as an example – there are seasonal variations in the air composition as well as a continuous change especially in the CO₂ content. Here is a four-component simplification of dry air:

Component	Molar ratio (mol/mol _{air})	Molar mass [4] (g/mol)	Molar mass in air (g/mol _{air})
N ₂	0.7808	28.013	21.872
O ₂	0.2095	31.999	6.7037
Ar	0.0093	39.948	0.3715
CO ₂	0.0004	44.01	0.0176
Total	1		28.965

The molar mass of air is 28.965 g/mol.

Other gas mixtures can be calculated using a similar method.

The chemical formula of water is H₂O, which means that each molecule contains one oxygen and two hydrogen atoms.

Component	Molar mass [4] (g/mol)
H ₂	2.0156
O	15.999
Total	18.0146

The molar mass of water is 18.0146 g/mol.

11. Conversions to imperial units

Fahrenheit temperature: $T_{°F} = T_{°C} \cdot \frac{9}{5} + 32$

psi (pounds per square inch): $p_{psi} = 6894.76 \cdot p_{Pa}$

Absolute humidity in grains per cubic feet: $A_{gr/ft^3} = 2.2883 \cdot A_{g/kg}$

Mixing ratio in grains per pound: $x_{gr/lb} = 0.14286 \cdot x_{g/kg}$

Enthalpy in BTU per pound: $h_{BTU/lb} = 2.324 \cdot h_{g/kg}$

Note that zero enthalpy has a different base temperature: 0 °F is often used together with the Btu/lb unit.

12. References

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- [3] B. Hardy, “Formulations for vapor pressure, frostpoint, temperature, dewpoint temperature, and enhancement factors in range -100 °C to 100 °C,” in *Papers and abstracts from the Third International Symposium on Humidity and Moisture, vol. 1*, Teddington, 1998.
- [4] J. Meija, T. B. Coplen, M. Berglund, W. A. Brand, P. De Bièvre, M. Gröning, N. E. Holden, J. Irrgeher, R. D. Loss, T. Walczyk and T. Prohanska, “Atomic weights of the elements 2013,” IUPAC, 2015.

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