Control Theory

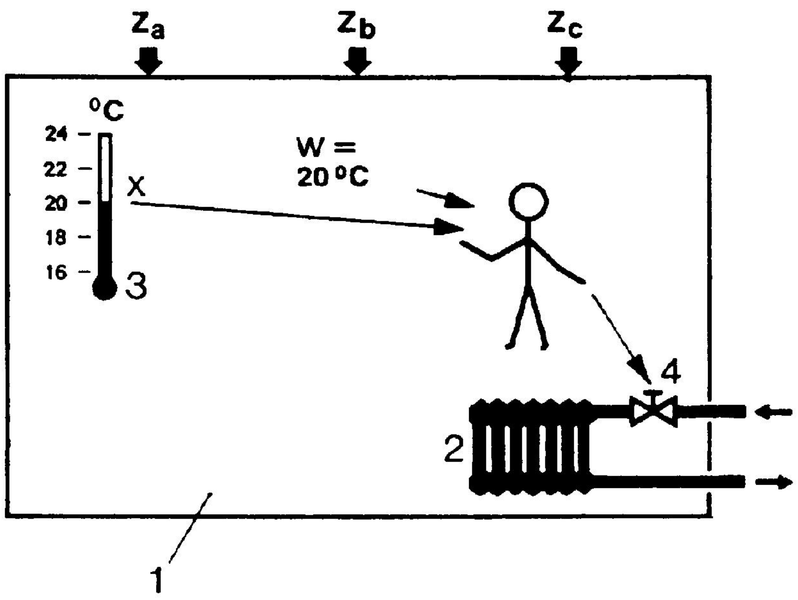
Introduction

- The purpose of automatic HVAC system control is to modify equipment performance to balance system capacity with prevailing load requirements.
- All automatic control systems do not employ the same kind of control action to achieve this objective.
- Despite the many types of controllers and control devices available in the market they all follow common fundamentals of control theory.
- This course will examine the control functions found in HVAC systems and explain the different applications where they are applied.
- Control terminology and definitions that are used in the explanations and examples of the various control functions will be explored.

Control Theory

Manual Control

- The most basic control function is manual control, for example manual heating control.
- The control task is to maintain a constant room temperature in room (1) having radiator (2).
- The desired room temperature is 20°C and the operator must keep the temperature constant.
- He compares the measured temperature (x) with the desired temperature (w).
- He opens or closes the valve to increase or decrease heating as required.
- Disturbances Za, Zb, Zc, influence the heating requirements, eg. heat transfer through walls, etc.



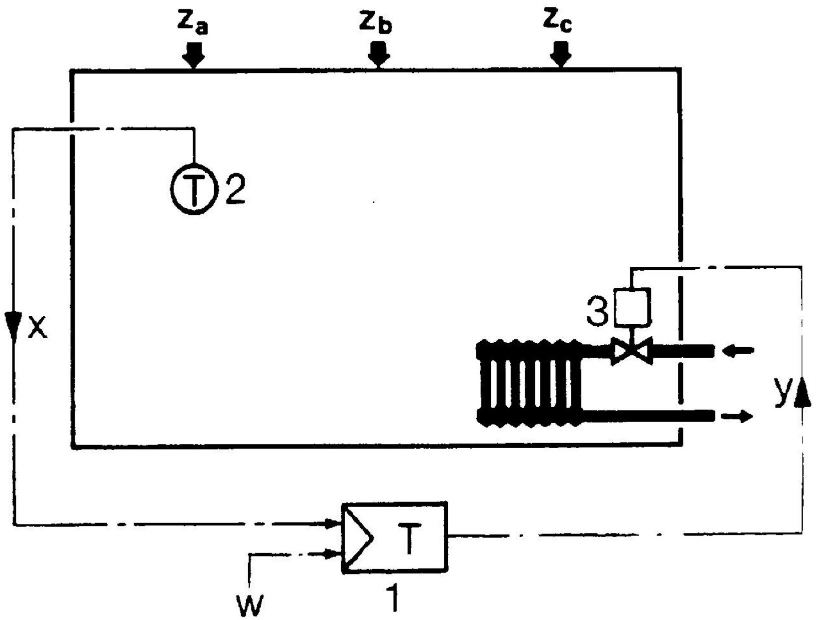
1. Room
2. Radiator.
3. Thermometer.
4. Manually operated valve.

(w) Desired room temperature (reference. value w)
(x) Measured room temperature(controlled variable x)
Za, Zb, Zc, Disturbance values.

Control Theory

Automatic Control

- The operator is replaced by a temperature controller (1) to maintain room conditions.
- The measured room temperature is called the the controlled variable X sensed by a detector (2)
- The desired room temperature is referred to as reference value W (controller setpoint) .
- The difference between the controlled and measured values is control deviation XW (X - W)
- The controller calculates the difference X and W, and positions the heating valve.
- The controller closes or opens the valve relative to control deviation (X - W) via correcting variable Y.



1. Temperature controller.
2. Temperature detector.
3. Control valve with actuator.

(w) Reference value (setpoint)
(x) Controlled variable (measured temperature value)
(y) Correcting variable (output to control valve)
Za, Zb, Zc, Disturbance values.

Control Theory

Controlled System Essentials

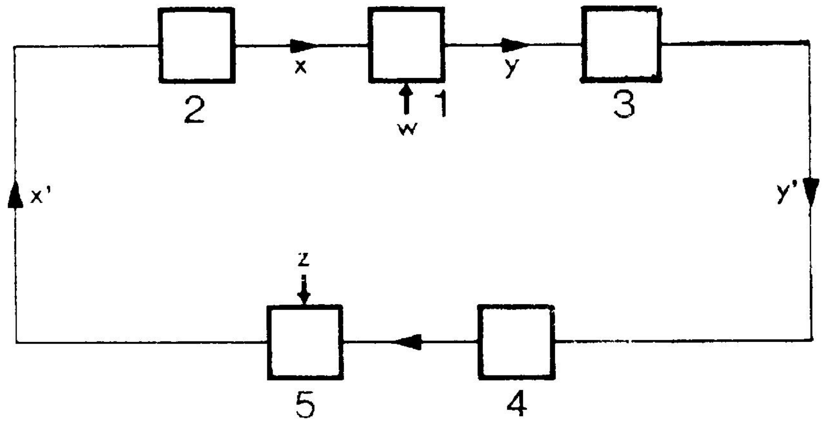
- In the control process the controller must know to what extent the valve must be opened or closed for a given control deviation. In many cases it is necessary to amplify the control deviation.
- It is important to know how the controller should intervene in the case of a control deviation e.g. rapidly or slowly. The reaction to a control deviation is known as the controller time response.
- The following tasks have to be fulfilled in a control system,

1. Measurement.
2. Comparison.
3. Amplification.
4. Generation of a time response.
5. Positioning.
6. Measurement.

Control Theory

Closed Control Loop

- The block diagram illustrates a typical control loop, and in this case , a closed loop.
- Controller (1) has a set point (W) which is the desired temperature in the room.
- The temperature detector (2) senses a change in room, and a control deviation (X - W) is created.
- The output signal (Y) opens the heating valve to adjust the room temperature to set point value.
- The radiator (4) heats up the room (5), temperature increases, and the detector (2) senses the change.
- In such a block diagram, it is very easy to recognise the closed loop and unidirectional flow.

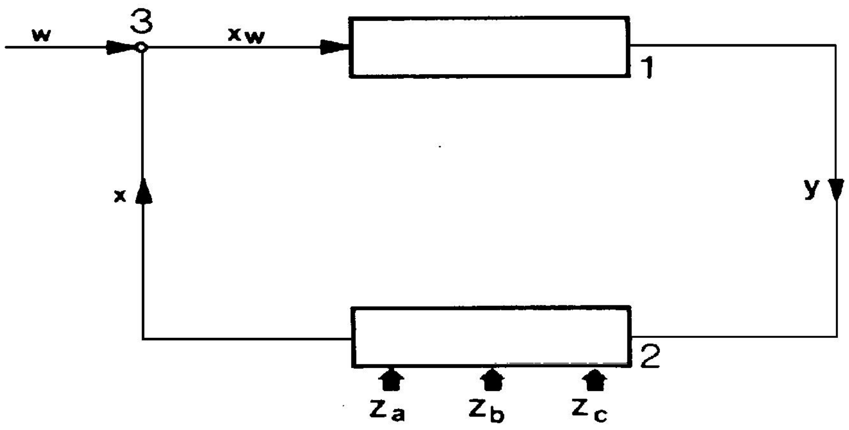


1. Controller.
2. Temperature detector.
3. Control valve actuator.
4. Radiator.
5. Room

Control Theory

Closed Control Loop

- The block diagram illustrates a simplified closed loop block diagram.
- We find that the control loop consists of two main groups of control loop units.
- Control device (1) consists of the temperature detector, controller, and control valve.
- Control device is that part of the loop that controls the process, and delivered by the controls supplier.
- Controlled system (2) consists of the radiator and room .
- Controlled system is that part of the loop that is controlled, and delivered by the plant manufacturer.



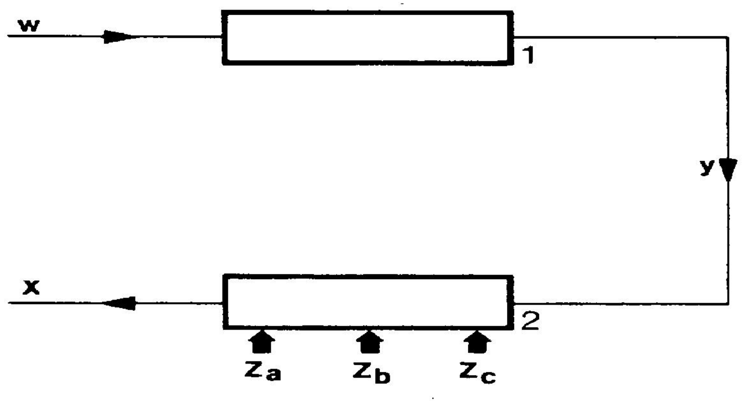
1. Control device.
2. Controlled system.
3. Summation point, comparator (X - W)
4. Radiator.

XW. Control deviation.

Control Theory

Open Control Loop

- The block diagram illustrates a simplified open loop block diagram.
- The open control loop has no feedback from the controlled variable (X) to the control device (W).
- The desired set value and actual measured value are not compared to each other.
- Control device (1) consists of the temperature detector, controller, and control valve.
- Controlled system (2) consists of the radiator and room .
- The result of the positioning of the valve by the control action of the controller is not supervised.



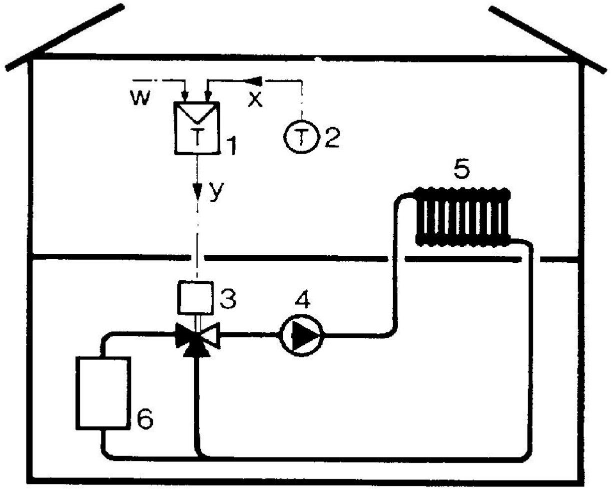
1. Control device.
2. Controlled system.

W. Reference value.
X. Controlled variable.
Y. Correcting variable.

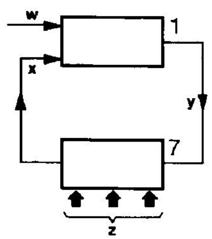
Control Theory

Closed Loop Example - Room Temperature Control

- Room temperature control to provide heating via a boiler and radiator.
- The room temperature is measured by means of the temperature detector (2).
- Value (X) is passed onto the controller and compared to the reference value (W).
- A control deviation will open or close the valve depending on the polarity of the deviation.
- The valve will continue to operate until the measured and desired values are equal.
- The output of the radiator gives feedback to the temperature detector.



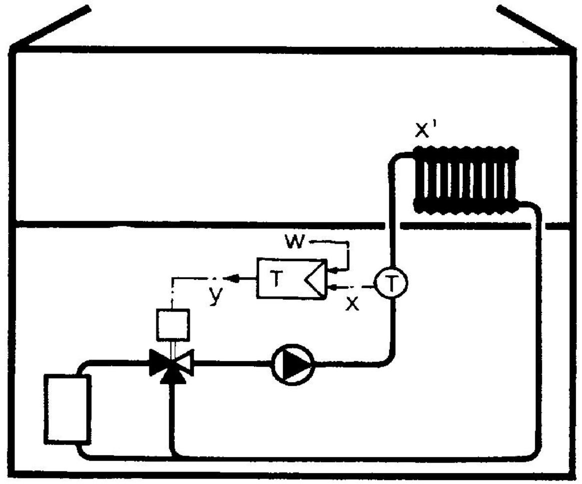
1. Temperature controller.
2. Temperature detector.
3. Mixing valve.
4. Circulation pump.
5. Radiator.
6. Boiler.
7. Controlled system.



Control Theory

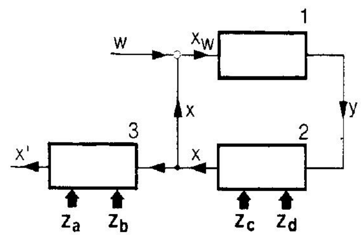
Closed & Open Loop Example - Flow Temperature Control

- This example contains both an open and closed loop control.
- The flow temperature is controlled in a closed loop controlled system.
- This is because the flow temperature (X) is fed back to controller input and compared to value (W).
- The room temperature is controlled in an open loop controlled system.
- The measured value (X') of the room temperature is not fed back to controller input.
- Therefore the flow temperature control is closed loop and room temperature control is open loop.



1. Control device.
2. Controlled system (closed loop)
3. Open loop system.

X. Controlled variable.
X' Feed forward controlled variable.



Control Theory

Control Loop Units

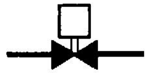
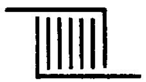
- We have seen already that each control loop unit can be represented by a simple box. (slide 5)
- These were designated as controller, detector, heating valve, etc. and are easily identified.
- It is the relationship between the input and output values for each control unit that is important.
- The control loop is always unidirectional and each box has an input and an output.
- A typical control loop unit as a block is shown with an input (e) and output (a).
- The output of one control loop unit block becomes input of the next control loop unit block.

Control loop unit as a block
e input
a output

Control Theory

Control Loop Unit Inputs and Outputs

- The inputs and outputs of the different boxes in the example on slide 5 are shown.
- The temperature detector box has an input of temperature and an output of electrical resistance.
- The resistance becomes the input to the controller and the output is a variable 0..10vdc voltage.
- The voltage becomes the input to the valve and the output is variable quantity of warm water.
- The quantity of warm water becomes the input to the radiator and the output is heat energy.
- The heat energy becomes the input to the room, and the output the temperature in the room.

Input	e → Control loop unit → a	Output
Temperature	T	Resistance
Resistance		Voltage 0 ... 10 V=
Voltage 0 ... 10 V=		Quantity of warm water
Quantity of warm water		Heat energy
Heat energy	Room	Temperature
Controlled variable x, respectively, control deviation x_w	Control device	Correcting variable y
Correcting variable y	Controlled plant	Controlled variable x

Control Theory

Controlled System.

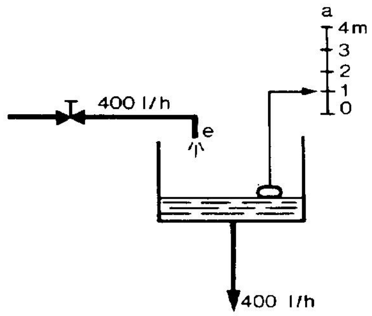
- The controlled system is that part of the control loop which has to be controlled.
- The details of the controlled system need to be evaluated to select the control device required.
- The quality of the control can only be realised by knowing the behaviour of the controlled system.
- Controlled systems can be considered as control loop units with the correcting unit y as the input e and the controlled variable x as the output a.
- There are two types of controlled systems :
  - Unbalanced controlled systems
  - Balanced control systems.

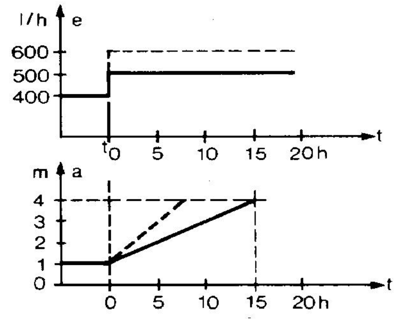
Block diagram of a controlled system.
1 Regulating unit
2 Controlled system
e input
a output
x Controlled variable
y Correcting variable

Control Theory

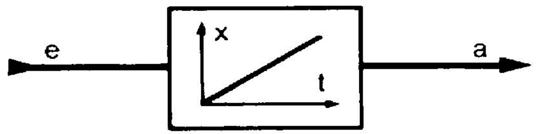
Unbalanced Controlled Systems

- This level control is an example of an unbalanced controlled system. (integral behaviour)
- The input value e is the quantity of water l/h, and output value a is the level of water m in the tank.
- The output remains constant at 1m, if the amount of supply water is equal to the amount of discharge water.
- If at time t_o, the quantity of supply water is increased the water level increases linearly with time.
- After a step change in the input value, the output changes continuously but does not reach a new steady state.
- The rate of change of the output value (level) is proportional to the change of the input value.





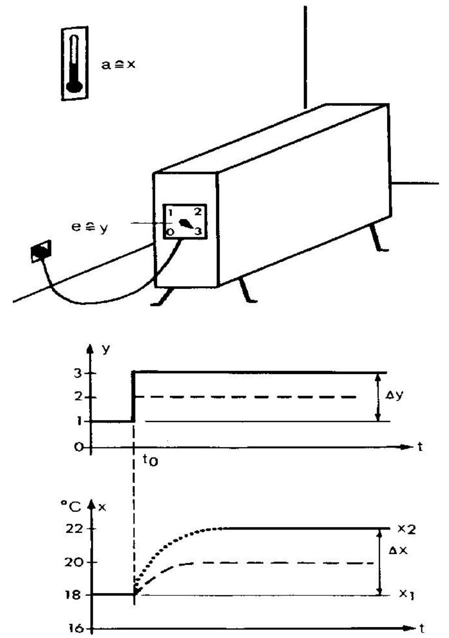
e Input
a Output
t Time
t_o starting point



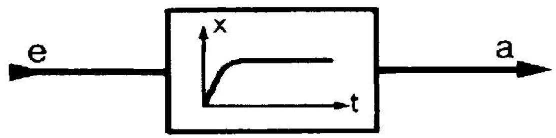
Control Theory

Balanced Controlled Systems

- Following a change in the input, the output value always attempts to attain a new stationary status.
- In this type of control system a new state of balance is attained (proportional behaviour)
- An electric heated radiator with a 3 stage switch illustrates the working of the balanced system.
- If the heating output is increased from stage 1 to 2, the room temperature increases to a new level.
- If the heating output is increased from stage 2 to 3, the room temperature increase is doubled.
- The change r x in the output value is proportional to the change r y in the input value.

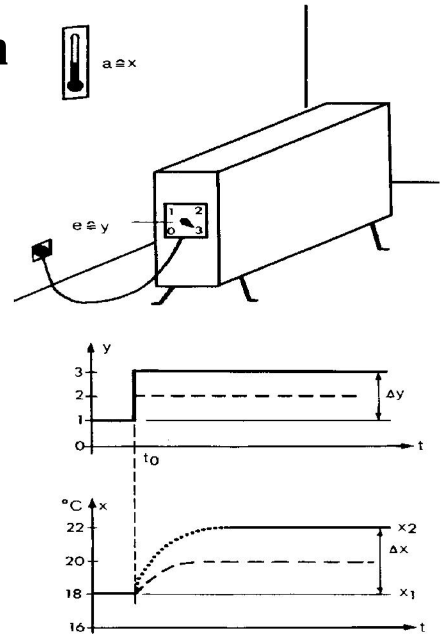


x1 Initial steady state
x2 New steady state
r x Change in value of the controlled variable
r y Change in the value of the output value
T_0 Starting point.



Control Theory

Static Behaviour of the Balanced Controlled System



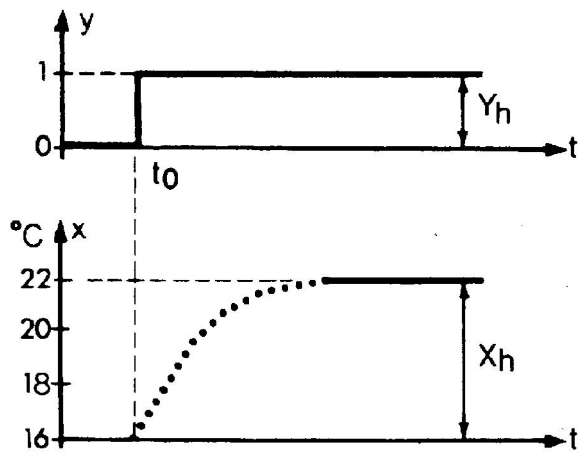
- The term “static behaviour of a balanced controlled system ” implies the relationship between the output value **a** (controlled variable x) and the input value e (correcting variable y) in the stationary state.
- If we look at the electric heated radiator, a change in value of the correcting variable **y** (switching from pos. 1 to 3) has resulted in a specific change in the value of the controlled variable **x** (increase in room temperature from 18°C to 22°C (r x = 4 K) .
- Proportional control factor KS = $\dfrac{\text{change r x of the controlled variable}}{\text{change r y of the correcting variable}}$
- For this example, KS = $\dfrac{\text{r x}}{\text{r y}} = \dfrac{4\text{ K}}{2\text{ steps}}$ = 2 K per step.

x1 Initial steady state
x2 New steady state
r x Change in value of the controlled variable
r y Change in the value of the output value
T_0 Starting point.

Control Theory

Static Behaviour of the Balanced Controlled Systems

- If the step switch of the radiator is turned directly from pos. 0 (off) to the highest possible change step 3, this will result in the maximum possible change of the room temperature under the momentary conditions.
- This means , a change in the correcting variable **y** by the correcting range Yh of the correcting variable results in a change of the controlled variable **x** by the controlled range Xh of the controlled variable.



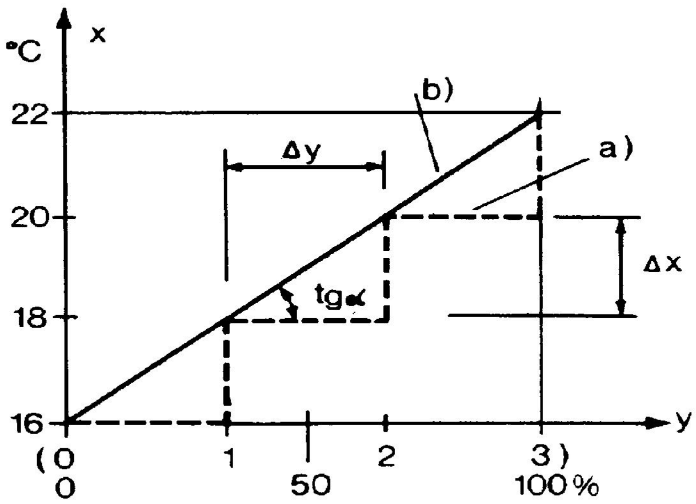
Change in the room temperature for a unit step change from 0 to 1.
Yh Correcting range of the correcting variable.
Xh Control range of the controlled variable.

- If r y is put as the unit step change (= Yh) from 0 to 1, in the equation Ks = r x / r y, this will lead to a r x which corresponds to the control range Xh.
- $Ks = \frac{r\ x}{r\ y} = \frac{Xh}{Yh} = \frac{6\ K}{1} = 6\ K$ (control range Xh)

Control Theory

Static Behaviour of the Balanced Controlled Systems

- The relationship between the change r x of the controlled variable and the change r y of the correcting variable can be plotted on a graph.
- This gives the **static characteristic**, also known as the **control characteristic** of the controlled system.
- Let us imagine the 3 steps of the radiator has an infinite number of steps, each stage of the step switch leads to a corresponding percentage change in the room temperature.
- Thus, in this example, the control characteristic is linear. It's slope r x / r y = tan α corresponds to the proportional control factor. see slope (b)
- However, in practice, the control characteristic in most cases is not linear.



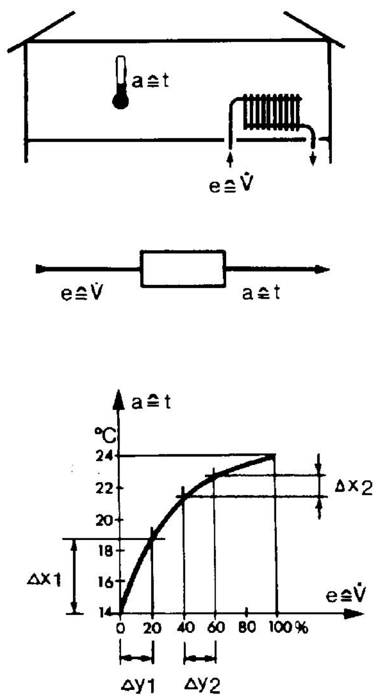
Control characteristic of the electric radiator.

a) 3 - stage control Xh
b) Continuous control

Control Theory

Static Behaviour of the Balanced Controlled Systems

- The static characteristic of a room heated by a hot water radiator illustrates the non linear behaviour.
- In this case, the input value **e** is the volume of flow **V** of the hot water at constant temperature and the output value **a** is the room temperature **t**.
- If the relationship between **V** and **t** is plotted on a graph, the result will be the control characteristic of this controlled system.
- As we can see, this control characteristic is not linear, i.e. the change r y1 between 0…20% in the flow volume causes a change r x1 in the room temperature from 14°C to 19°C, i.e. by 5K
- The change r y2 between 40.. 60% results in a change r x2 in the room temperature of only 1.4K.



Control characteristic of a radiator room heating system with variable water flow.

Control Theory

Dynamic Behaviour of the Balanced Controlled Systems

- The term “ dynamic behaviour of a controlled system “ implies the relationship between the change r x of the output value and the change r y of the input value in function of time.
- In a controlled system, the output value corresponding to a given change of the input value is generally attained only after a certain period of time.
- This time lag can be caused by the flow or transport time, or by the storage behaviour (storage of energy) of the control loop units .
- The flow or transport time lag is related to the heating or chilled water or air producing the required result to satisfy the control system. It is the time it takes the change in prime energy source to reach the final measuring point. It is called the dead time of the system.
- The storage of energy (charging of storage containers) examples are as follows,
  - Heating an electric hot plate by means of an electric current.
  - Heating a room by a radiator.
  - Filling a pressure tank with air.

Control Theory

Dynamic Behaviour of the Balanced Controlled Systems

- In practice we generally come across controlled systems in which two or more storage containers are connected in series (electric current heats the electric hot plate, the electric hot plate heats the water in the pan.)
- Controlled systems can be grouped according to the number of storage containers in the system,
  - Controlled systems without any storage containers } Single storage container systems
  - Controlled systems with one storage container } Multiple storage container systems
  - Controlled systems with multi storage containers } Multiple storage container systems
- The controlled systems can also be classified according to the order of the corresponding differential equation, the latter being equal to the number of the storage containers connected in series:
  - Controlled systems without any storage container } Controlled systems of zero order.
  - Controlled systems with one storage container } Controlled systems of first order.
  - Controlled systems with two or more containers } Controlled systems of higher orders
- Besides it’s static behaviour, the order of the controlled system (number of storage containers) mainly determines the degree of difficulty of a control task.

Control Theory

Evaluation of a Controlled System by the Step Function Response Method.

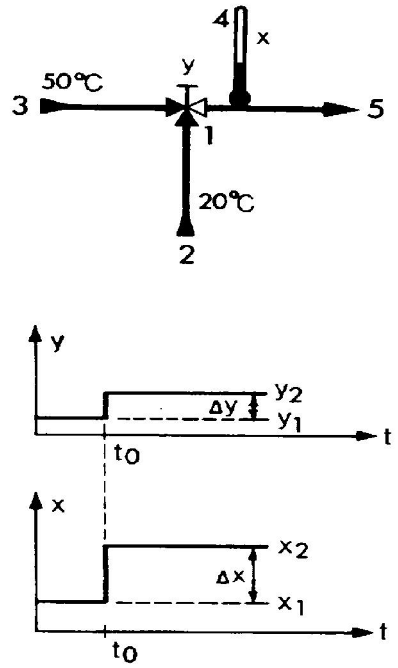
- This method of evaluating the time behaviour of a controlled system can be used quite simply by the practical engineer, and therefore it is more commonly employed than other methods.
- The **step function response** of a controlled system is obtained by suddenly changing the input value (correcting variable y) by an arbitary amount (step function) and then plotting the changes in the output value (controlled variable x) in function of time.
- The curve obtained thereby shows the **transfer behaviour** of the controlled system.
- If the step function response refers to a change of input value from 0… 100% (0 … 1) , one would obtain the **transfer function** of a controlled system.
- The resultant transfer function not only shows the time response of the controlled system, but also the control range Xh of the controlled variable , or - assuming a linear control characteristic - the proportional control factor Ks of the system.

Control Theory

Controlled Systems without any Storage Container.

(controlled systems with zero order.)

- To illustrate this type of controlled system , let us consider a part of a domestic hot water plant.
- In this example the correcting unit is a manually operated valve 1which is supplied with cold water 2 and hot water 3, and supplies mixed water 5 at the tap.
- A fast response thermometer 4 before the tap to measure the mixed water temperature (controlled variable x)
- The position of y1 of the manually operated valve gives a corresponding temperature x1 of the mixed water.
- The change r x in the controlled variable occurs practically without ant time lag in relation to the change r y of the correcting variable.



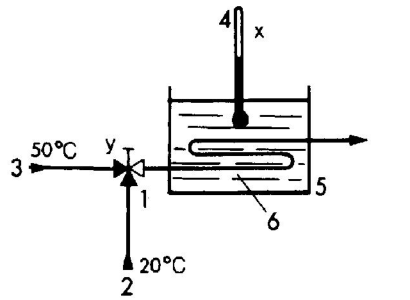
1. Manually operated valve.
2. Cold water.
3. Hot water.
4. Thermometer.
5. Tap, mixed water.

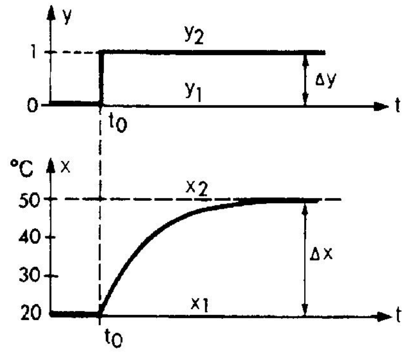
Control Theory

Controlled Systems with one Storage Container.

(controlled systems of first order.)

- To illustrate this type of controlled system , let us consider the heating of a hot water container.
- The liquid in the container **5** (storage container) is heated (charged) by the hot water **3** which flows through the coil of pipe **6** inside the container.
- The input value of the controlled system (of the storage container) is the position **y** of the manually operated valve **1** and the output value is the temperature **x** of the liquid which is measured by the thermometer **4**.
- The position of y1 of the mixing, only cold water **2** flows through the container giving an assumed temperature x1 = 20°C
- If at time t0 the valve is positioned so that only hot water flows, the rise in temperature is relatively quick at the beginning, but becomes increasingly slower until the final temperature 50°C.





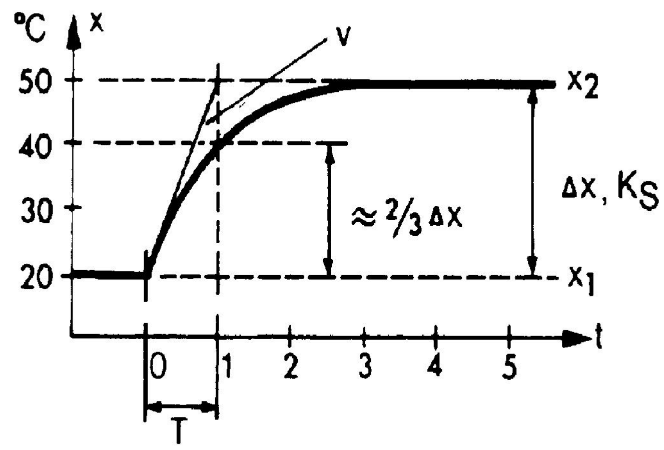
1. Manually operated valve.
2. Cold water.
3. Hot water.
4. Thermometer.
5. Container for the liquid.

Control Theory

Controlled Systems with one Storage Container.

(controlled systems of first order.)

- In the previous example, the heating - up time (charging time of the storage container) depends solely upon the quantity of liquid (size of the container)
- Smaller quantity - Short heating - up time
  Larger quantity - Long heating - up time.
- Irrespective of whether heat, pressure or electricity is stored a step change of the input value always results in the same charging characteristic (exponential function.)
- Time constant T is a function of the storage capacity and and the proportional control factor KS.
- The time constant T is the time which the output value - maintaining it's initial rate of change - will change by r x corresponding to the change the r y of the input value.



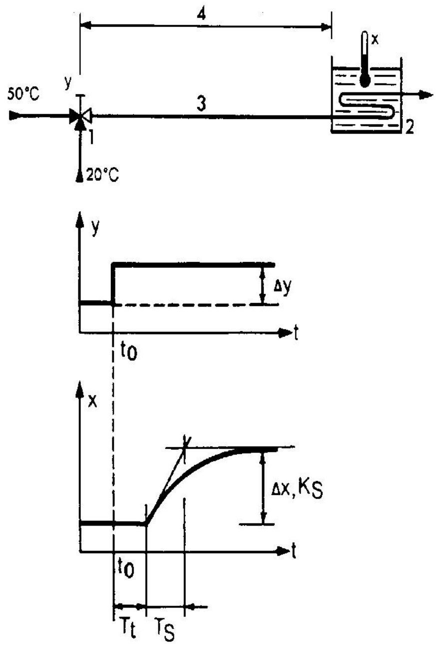
Charging characteristic of a storage container and the time constant
v Initial rate of change
T Time constant

Control Theory

Controlled Systems with one Storage Container and Dead Time.

(controlled systems of first order.)

- This example is of a controlled system with one storage container and dead time.
- It is the same plant as before, but in this case, the manually operated valve 1 has been mounted some distance away from the liquid container 2.
- At time t0, the valve is positioned to increase hot water in supply pipe 3 leading to the container.
- The temperature x of the liquid in the container can start to increase only when the hot water has been transported to the coil of the pipe.
- This period of time is known as the dead time Tt. After the dead time has elapsed, the controlled variable x changes in the usual manner for a controlled system with one container.



1. Manually operated valve.
2. Liquid container with a coil .
3. Supply pipe.
4. Transportation distance.

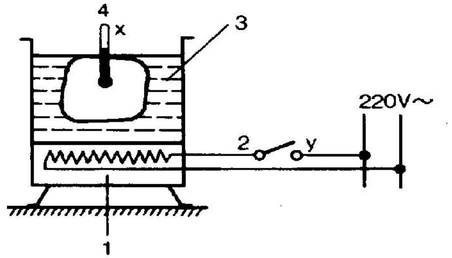
Tt. Dead time.
Ts. Time constant.

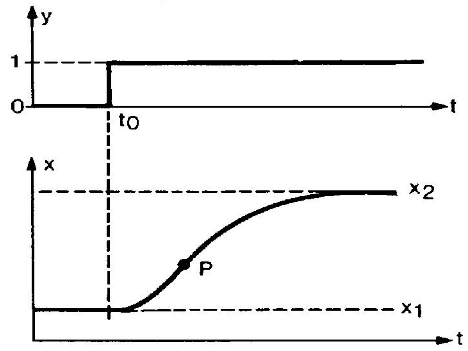
Control Theory

Controlled Systems with more than one Storage Container

(controlled systems of higher order.)

- We generally come across controlled systems in which we have two or more storage containers.
- Examples are : - Heating plants with valves, hot water flow, pipe, radiator, room.
  - Ventilating plant with valves, heater battery, supply air duct, room.
- This example is of a typical household application of an electric hot plate heating a cooking pan.
- The three storage containers are the hot plate, water, and the substance to be cooked.
- The curve starts with a horizontal tangent, thereafter it rises slowly at first, and then more rapidly until it reaches turning point P, which always lies below half of the final value of the new stationary state.





1. Electric hot plate.
2. Switch.
3. Cooking pan with water.
4. Thermometer.

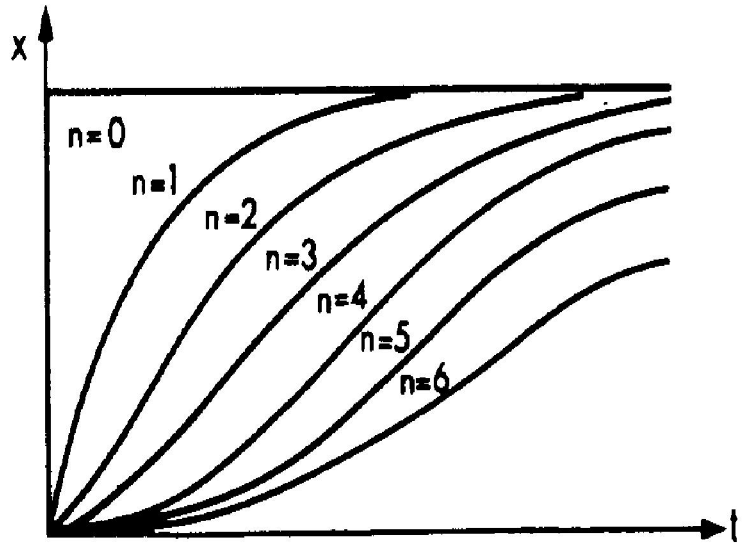
x1, x2. Steady states.
P. Turning point.

Control Theory

Controlled Systems with more than one Storage Container

(controlled systems of higher order.)

- In the case of controlled systems of higher order, the highest rate of change of the controlled variable is to be found at the turning point P (change of direction) of the step function response S - curve.
- This diagram shows the step function response of controlled systems which may comprise up to n = 6 similar storage containers.
- It can been seen that the higher the order of the controlled system, the smaller is the slope of the step function response.
- The horizontal tangent and the slow rise to the theoretical point P has longer delay time as the number of storage containers increases.

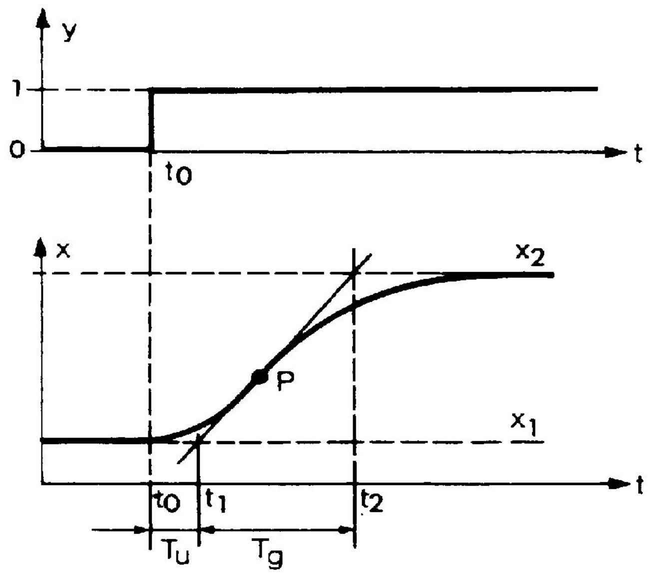


n Number of similar storage containers.

Control Theory

The Degree of Difficulty of a Controlled System.

- In a controlled system of higher order, the transient behaviour (step function response) is characterised by the delay time Tu and the compensating time Tg.
- The controllability is expressed by the degree of difficulty λ and is derived from the relationship between the delay time Tu and the compensating time Tg.
- This tangent through the turning point intersects the two steady states at x1 and x2.
- t1 and t2 identify the where the tangent intersects the time axis.
- - Time difference t0 … t1 = Delay time Tu
  - Time difference t1 … t2 = Compensating time Tg



Tu delay time
Tg compensation time

Control Theory

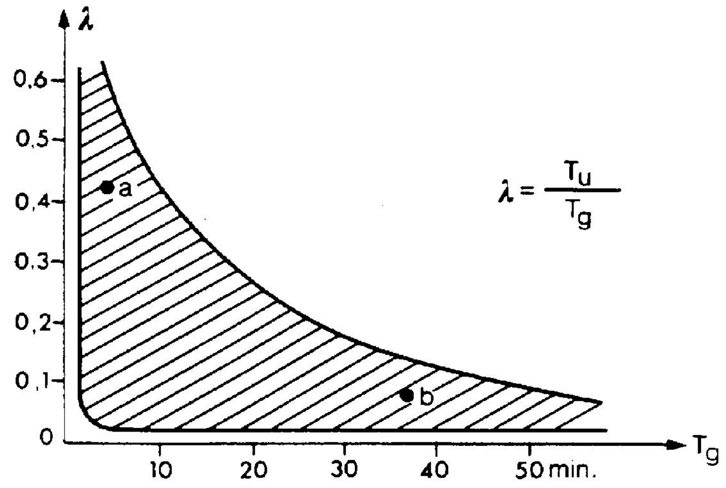
The Degree of Difficulty of a Controlled System.

- The control engineer is specially interested in the controllability of a system.

- The controllability is expressed by the degree of difficulty λ and is derived from the relationship between the delay time Tu and the compensating time Tg.

- Degree of difficulty $\lambda = \dfrac{\text{Delay time}}{\text{Compensating time}} = \dfrac{Tu}{Tg}$

- In HVAC systems, controllability can be classified as,
  λ < 0.1 : Easily controllable system.
  λ 0.1 … 0.3 : The system is less easy to control.
  λ > 0.3 : The system is difficult to control.



λ Degree of difficulty
Tu Delay time
Tg Compensating time
a A fast reacting system, but dificult to control
b A slow reacting system, but easy to control

Control Theory

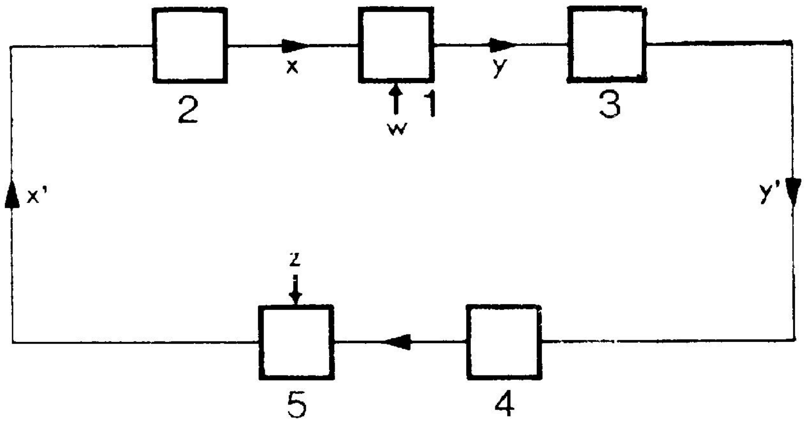
Evaluation of a Controlled System by the Frequency Response Method.

- The alternative to the step function response method to evaluate a controlled system is the **frequency response method**. Sinusoidal oscillations with constant amplitude but variable frequency are used instead of the step change.
- If these input oscillations are not fast enough for the controlled system, the output variable, i.e. the controlled variable x, will also oscillate in the same rhythm. These oscillations of the controlled variable represent the oscillatory response.
- If the frequency of the input oscillations is changed, i.e. lowered or raised, the same frequency will also appear after some time at the output of the controlled system. The amplitude of the output oscillation and its phase relationship in reference to the input oscillation will change simultaneously.
- The amplitude and phase relationship of the input versus the output, and plotted to identify the controlled system characteristics.
- The frequency response method has a much higher control resolution potential than the step function response, but is also much more complex. For the practical engineer, the step function response method for the evaluation of the dynamic behaviour of a control loop unit is easier to understand, and is quite suitable to analyse control loops in HVAC systems.

Control Theory

Controllers.

- The function of a controller **1** is to cause an automatic change of the correcting variable **y** in order to eliminate the control deviation **xw** which arises from a change in reference value or disturbance.
- To be able to fulfil this task, the controller must continuously measure (detector 2) the value of the controlled variable and compare it with set value **w**.
- Depending upon the result of this comparison, the position of a suitable correcting unit is changed in such a way as to eliminate the control deviation, i.e. the value of the controlled variable again equals the set value.

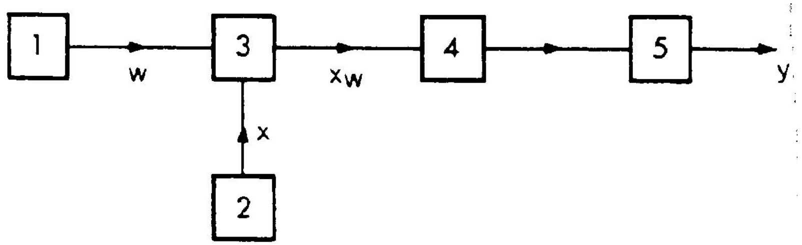


1. Controller.
2. Temperature detector.
3. Control valve actuator.
4. Radiator.
5. Room

Control Theory

Layout of a Controller.

- Setting unit **1** for setting the desired value **w** of controlled variable (remote or inbuilt set point adjustment.)
- Detector 2 for measuring the controlled variable x (the signal can be current, voltage, or pneumatic)
- Comparator 3 for forming the difference between the measured and set values of the controlled variable i.e. formation of the control deviation xw = x - w.
- Amplifier 4 for the amplification of the signal given by the comparator 3 (e.g. electronic amplifier) and for influencing the control behaviour (time response)
- Correcting unit 5 for changing the correcting variable y (e.g. control valve and actuator)



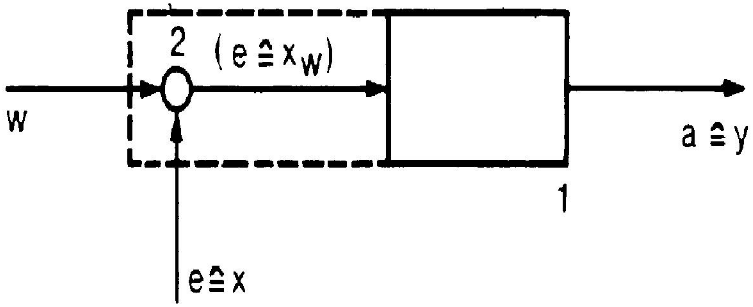
1. Setting unit.
2. Detector.
3. Comparator.
4. Amplifier.
5. Correcting unit.

w Reference value.
x Controlled variable.
xw Control deviation (x - w)
y Correcting variable.

Control Theory

Static and Dynamic Behaviour of a Controller.

- A controller can be considered as a control loop unit with control deviation xw as it's input value e and the correcting variable y as it's output value a.
- The controlled variable x is defined as the the input value e, provided the reference value w remains constant.
- To select the controller and achieve the control quality in conjunction with the controlled system, it is of decisive importance to know how far a controller will change the position of a correcting unit as a result of control deviation xw and how quickly it will act.
- Both these characteristics can be evaluated by recording the static and dynamic behaviour of the controller.



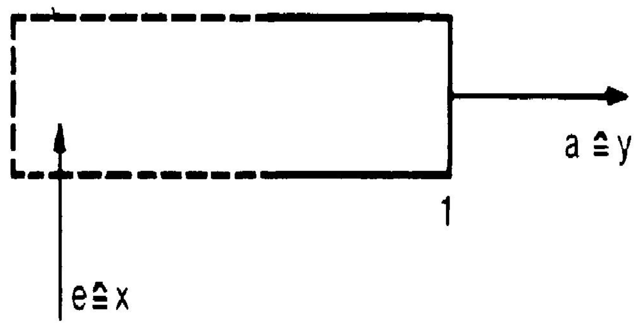
1. Controller.
2. Comparator.

e. Input
a. Output
5. Reference value
w Reference variable
x Controlled variable.
xw Control deviation
y Correcting variable.

Static Behaviour of a Controller.

- The term “ static behaviour of a controller “ implies the relationship between the output value a (correcting variable y) and the input value e (controlled value x) in the steady state.
- It is important to know how big the change r y of the correcting variable will be in relation to the change r x of the controlled variable.
- This ratio , the proportional control factor KR of the controller (also called amplification factor.)
- Proportional control factor KR

$$= \frac{\text{change in value of correcting variable } y}{\text{change in the value of the controlled variable } x} = \frac{r\ y}{r\ x}$$



1. Controller.
e. Input
a. Output
x. Controlled variable.
Y. Correcting variable.

Control Theory

Dynamic Behaviour of a Controller.

- The dynamic behaviour of a controller - also known as as the transfer behaviour or time response - shows the change in the output value of a controller (correcting variable y) in function of it's input value (controlled variable x) and time.
- In most cases the characteristics of a plant (Tu, Tg, KS) are pre-determined and cannot be influenced by the control engineer.
- The time behaviour of the plant controller must be adjusted to achieve an optimum control quality. This time behaviour must be produced artificially within the controller and is mostly adjustable.
- Step function response gives an indication of the time behaviour of a controller. To obtain a step function response, a step change of the input value by an arbitrary amount (e.g. adjust set point) and record the change in the output value in function of the time.
- The curve achieved thereby represents the dynamic behaviour (transfer behaviour) of a controller

Control Theory

Classification of Controllers.

- There are large number of different designs for controllers. They can be grouped together in a relatively small number of groups,
  - Type of controlled variable
  - Source of the energy for the correcting unit.
  - Control behaviour (type of change in the output signal)

- **Type of controlled variable** - .
  Some examples of the different controllers are,
  - Temperature controllers
  - Pressure controllers.
  - Humidity controllers
  - Universal controllers (these controllers can be adjusted to accept any controlled variable input (e.g. POLYGYR Joker)

Control Theory

Classification of Controllers.

- **According to the source of energy for the correcting unit.**
  - Controllers without auxiliary energy.
  - Controllers with auxiliary energy.
- **Controllers without auxiliary energy** depend on mechanical connection from the measuring to the correcting output of the controller. These controllers are called self acting or electro - mechanical controllers, some examples are, level controllers, thermostatic radiator valves, thermostats, etc.)
  The advantage of these controllers is that they work independently of any external energy supply, and are robust and less likely to fail. The disadvantage is they are for simple applications only.

- **Controllers with auxiliary energy** depend on outside energy sources to enable the measuring signal to be converted via an amplifier to operate the correcting unit.
  Depending upon the type of the auxiliary energy used, controllers are differentiated between,
  - Electric (electronic) controllers.
  - Pneumatic controllers.
  - Hydraulic controllers.

  Electric, electro-hydraulic, and electro-thermalactuators are used in combination with controllers using electricity as auxiliary energy.

Control Theory

Classification of Controllers.

- **According to control behaviour.**
  Generally controllers are placed in two main groups when classifying them according to the control behaviour (type of change of the output signal).
  - Modulating controllers.
  - Non modulating controllers.

- **Modulating controllers.**
  The modulating controller determines the direction of the deviation and the output correcting value changes continuously until it achieves a steady state of the controlled variable.
  Depending upon their time behaviour, the moduating controllers can be divided into these groups,
  - Controllers with proportional action … P - controllers.
  - Controllers with integral action … I - controllers
  - Controllers with proportional + integral action … PI - controllers
  - Controllers with proportional + differential action ... PD - controllers
  - Controllers with proportional + integral + differential action … PID - controllers

Control Theory

Classification of Controllers.

- **According to control behaviour.**

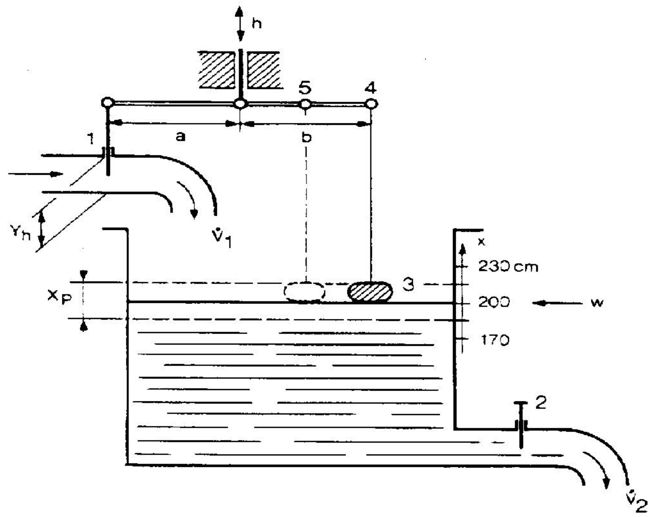
- **Non - modulating controllers.**
  The non - modulating controllers are characterised by the property that there output value, the correcting variable y can only take take two or more pre - determined values (e.g. 0, 33, 66, 100%) within the whole correcting range Yh and that it can be changed from one to the other value only in steps, i.e. in non - modulating mode.

- Depending upon the number of step positions which a controller of this type can control, they are differentiated as follows,
  - Single stage on / off controllers … (0…100%, on / off, open / close)
  - Two stage on / off controllers … (2 x on / off stages)
  - Four stage on / off controllers … (4 x on / off stages)
  - Multi - step controllers … (4..10 on / off stages)

Control Theory

Classification of Controllers … Modulating Controllers

- **Controller with proportional action (P - controller)**
  The working principle of a P - controller is explained by the mechanical water level control.
- The input value of the controller (controlled variable x) is the height of the water surface measured by the float.
- The output value of the controller (correcting variable y) is the position of valve 1 in the supply pipe.
- The load Q of the plant is the variable water discharge depending on the position of valve 2.
- The desired water level (set value w) is given by the height h at which the lever arm is fixed.
- The function of the control process is to see that the water level in the container does not change in spite of the fluctuations in the load, i.e., in spite of water discharge.



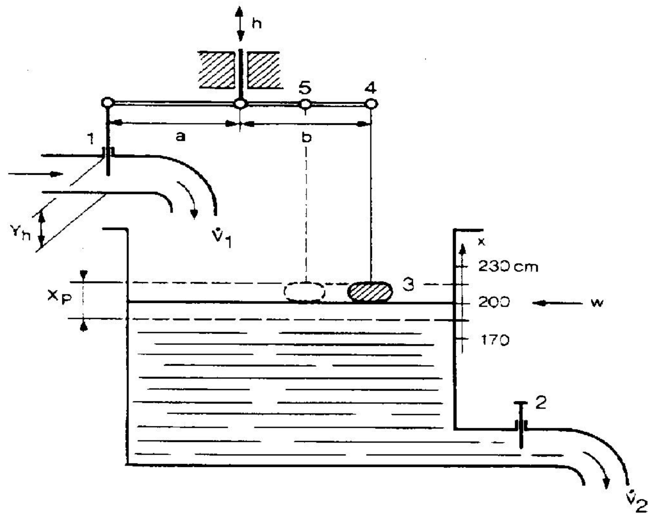
1. Valve in the supply pipe.
2. Valve in the discharge pipe.
3. Float gauge (detector)

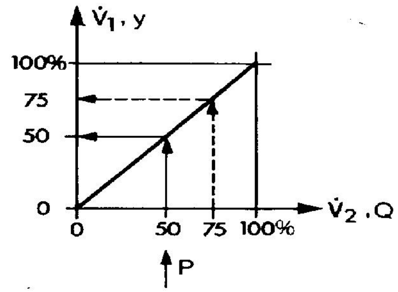
4,5. Fixing points for the float gauge
V1 Quantity of supply water
V2. Quantity of discharge water.
a+b Lever arm.
h. Height at which the lever arm is fixed.
x. Controlled variable (water level)
Xp Throttling range.
Yh. Correcting range of the correcting variable.

Control Theory

Classification of Controllers … Modulating Controllers

- **Controller with proportional action (P - controller) cont.** At constant supply pressure, the quantity V1 of the supply water is given by the position of the valve 1.
- Valve 2 is adjusted at mid position and the quantity of discharge water is half the maximum quantity.
- The float gauge 3 measures water level at 200cm (set value w), valve 1 adjusted to 50% open, with constant water level.
- Valve 2 is opened to 75%, the float falls to 185 cm, valve 1 opens to 75%, and water level remains constant again.
- This means that the value of the controlled variable x (water level) maintained will be different from that of the set value w. Thus the P - controller is load dependant.
- This remaining deviation from the set value is known as P-deviation or more commonly known as offset.





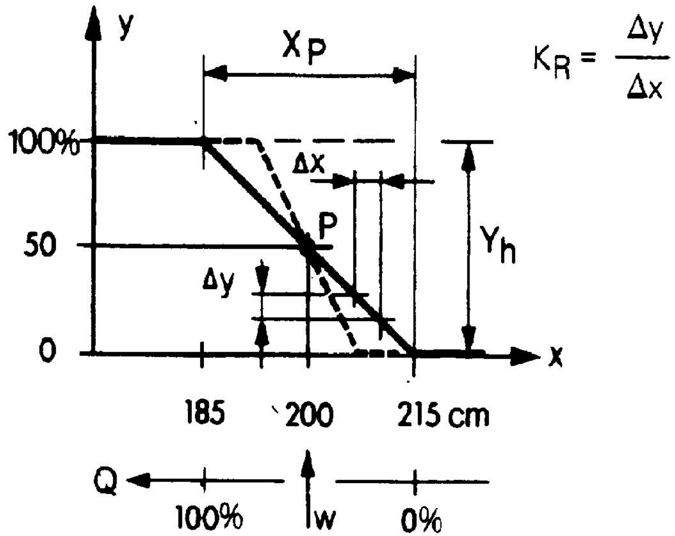
Control Theory

Static Characteristic of a P-controller.

- The previous example shows that the water level must change by a very specific range to enable valve 1 (correcting variable y) to attain it,s full travel Yh
- This range is known as throttling range Xp, or alternatively proportional band.
- The throttling range Xp is expressed in the units of the controlled variable x or in percent of the setting range.
- The water level with set value w (200cm) has to change by + - 15cm, and V1 can travel over it's whole range Yh.
- The reference range Wh is 60cm (170..230cm) so,

$$Xp\% = \frac{Xp\ (cm) * 100}{Wh\ (cm)} = \frac{30 * 100}{60} = 50\%$$

- If the float is connected to point 5, the throttling range Xp is halved, and V1 modulates over + - 7.5cm.

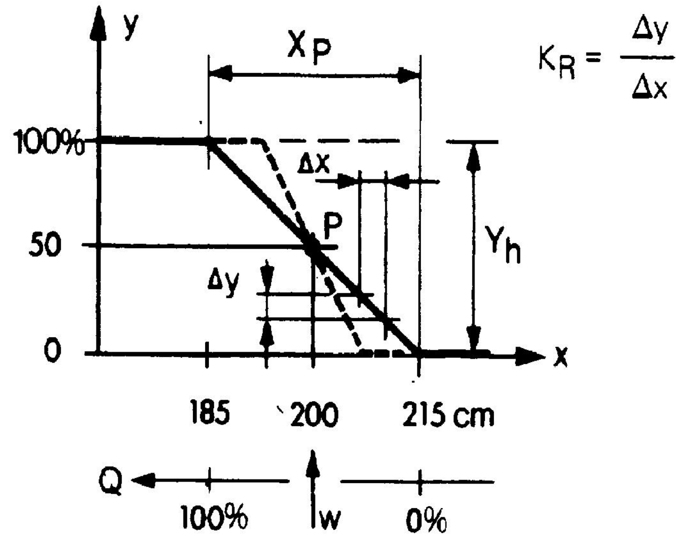


r x Change in the value of the controlled variable.
r y Change in the value of the correcting variable.
Xp Throttling range.
Yh Correcting range of the correcting variable.
Kr Proportional control factor.
Q Load.
P Calibration point.

Control Theory

Static Characteristic of a P-controller.

- The proportional control factor Kr of the controller indicates the change r x of the input value that results to a given change r y of the output value.
- Proportional control factor Kr = $\frac{\text{Change r y in the correcting variable}}{\text{Change r x in the controlled variable}}$
- This equation is also valid for the corresponding values Yh (correcting range of of the correcting variable) or Xh (control range of the control variable) and Xp.
- $Kr = \frac{r\ y}{r\ x} = \frac{Yh}{Xp} = \frac{Xh}{Xp}$
- The following is true for the proportional control factor: A large throttling range Xp will result in a small proportional control factor Kr and vice versa.
  Therefore $Kr = \frac{1}{Xp}$

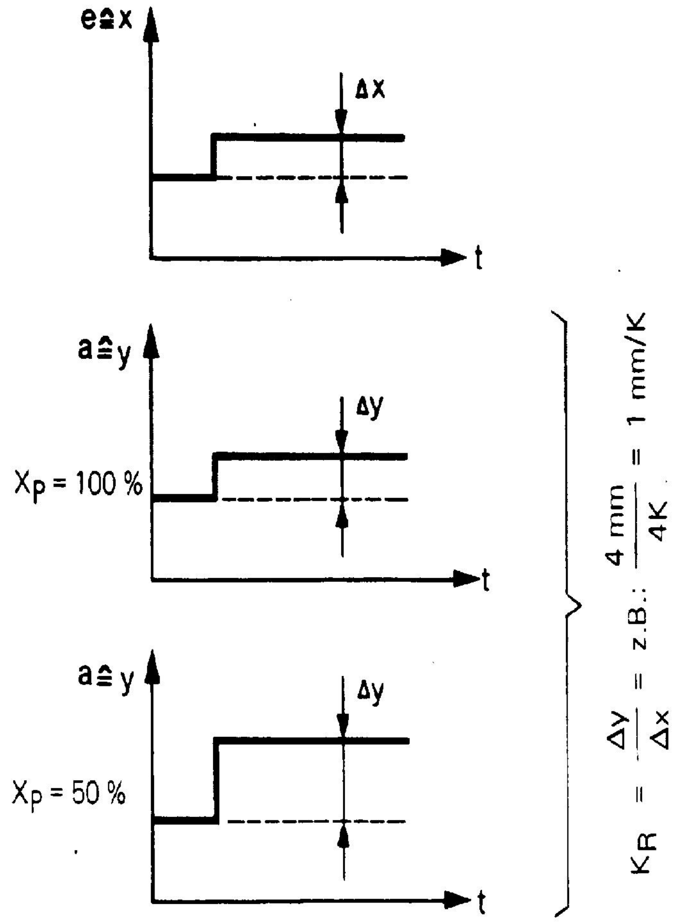


r x Change in the value of the controlled variable.
r y Change in the value of the correcting variable.
Xp Throttling range.
Yh Correcting range of the correcting variable.
Kr Proportional control factor.
Q Load.
P Calibration point.

Control Theory

Dynamic Characteristic of a P-controller.

- The dynamic behaviour of a P-controllercan be shown with the aid of the step function response.
- For a step change in the value of the controlled variable x, an ideal controller reacts with a step change in the correcting variable y in function of time.
- The magnitude of the change r y is a function of the change r x in the value of the controlled variable and the set value of the throttling range Xp.
- For a given step change, you can see for a Xp of 100% , the r y movement is half that of the Xp setting of 50%.
- Summing up, the higher the percentage Xp, the smaller the r y step function response.



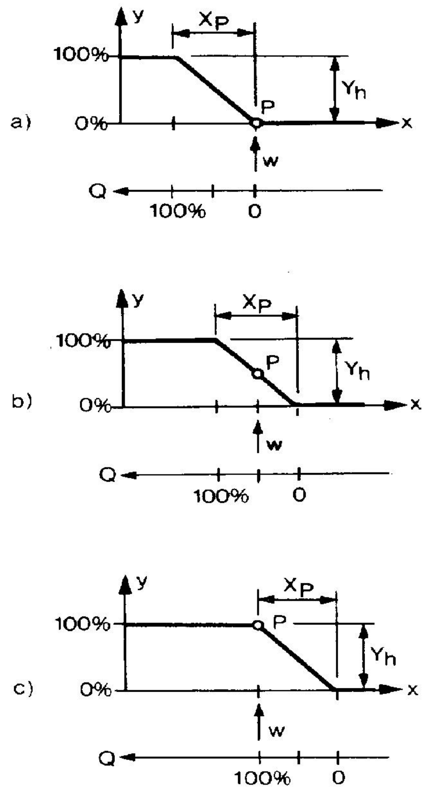
Control Theory

Dynamic Characteristic of a P-controller.

- The dynamic characteristic of a P-controller shows the following important features:
- The step function response has the same form as the input step function (proportional relationship between the input and output signals.)
- The corrective reaction of the controller takes place instantly; it's magnitude depends upon control deviation. Thus a P-controller is a fast reacting controller.
- The magnitude of the correction is limited (proportional to the deviation), i.e. the controller has inherent stability and therefore it can also give stable control of unbalanced controlled systems.
- The magnitude of the output signal in relation to the input signal can be set on the controller (throttling range Xp) and the controller can thus be tuned to the system to be controlled.

Shifting the Calibration Point.

- As in the water level control, , the set value w, in the case of of a P-controller , can only be maintained for a very specific load, the so called working point P.
- For any other load condition, there will always be a deviation, the so called steady state control deviation Xwb, or offset, the magnitude of which will always remain within the limits of the throttling range.
- In P-controllers it is possible to change the magnitude of the throttling range, as well to shift the calibration point P.
- When the control function is heating only, quite often the calibration point P is set to 50% so the offset is + - half Xp.
- When the control function is heating and cooling, the calibration point P is set to 0% of the heating Xp.

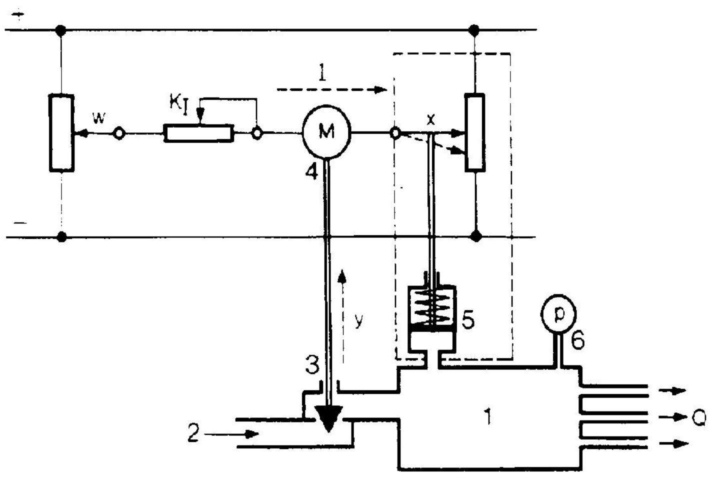


a) Calibration for load Q = 0%1.
b) Calibration for load Q = 50%
c) Calibration for load Q = 100%

Control Theory

Controller with Integral Action (I-controller)

- To explain an I-controller , we use a pressure control system. A pressure tank with supply pipe 2, valve 3, numerous air outlets (load Q), manometer 6, and pressure detector 5.
- The function of the control process is to maintain the tank pressure at a constant value in spite of the changing air discharge.
- When the tank pressure corresponds to the desired set value w, there will be no current through the actuator.
- Heavy discharge of air Q moves potentiometer in detector 5, and current flows in actuator, admitting more air at inlet 2.
- The magnitude of current I is adjusted by potentiometer KI, and the speed of the actuator increases with control deviation Xw. The valve will continue to open until deviation disappears.

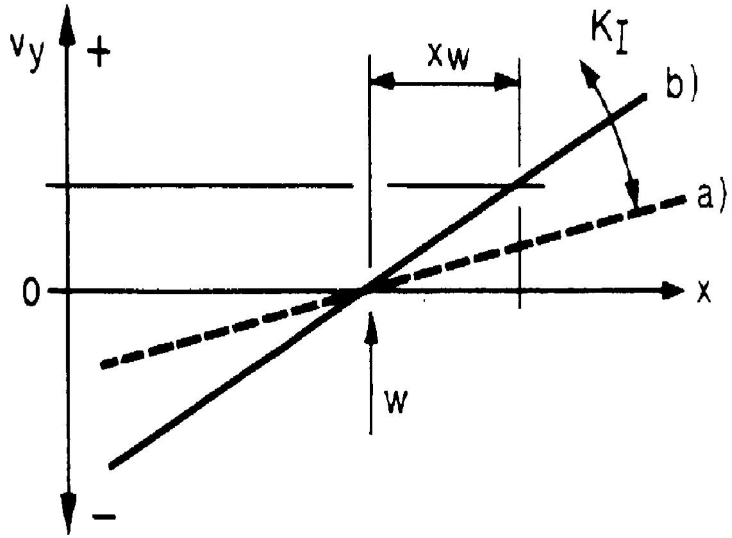


1 Pressure tank.	w Set value
2 Supply pressure	x Measured value
3 Control valve	y Correcting variable
4 Actuator	I Diagonal current
5 Pressure detector	KI Integral action function
6 Manometer	Q Load

Control Theory

Controller with Integral Action (I-controller)

- In a closed loop, the opening of valve 3 will cause the tank pressure to increase gradually .
- The control deviation Xw will again become increasingly smaller, and hence the floating rate Vy will become increasingly lower.
- The actuator will continue to run until the sliding contact of the pressure detector 5 has attained it's initial position.
- This means there is no more deviation, and no current flows through the actuator and the valve becomes stationary.
- Thus an I-controller eliminates the control deviation completely, it controls independently of load.

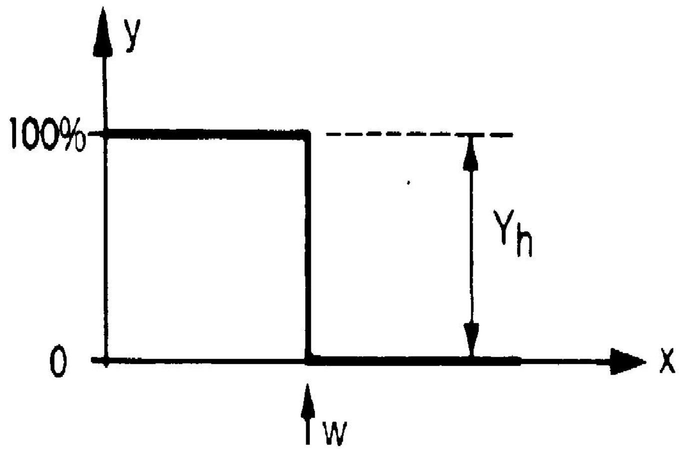


KI Integral action function
a) Low rate of change
b) High rate of change

Control Theory

Static Characteristic of a I-controller.

- In an I-controller, the correcting variable changes as soon as - or as long as - there is deviation between the set and the measured values.
- For any arbitrary control deviation Xw, and corresponding time period, any arbitrary value of the correcting variable y within the correcting range Yh can be attained.
- In the steady state, there is no direct relationship between the control deviation Xw, and the correcting value y as in the case of the P-controller.
- The diagram shows that the correcting variable is either 100% or 0% when the deviation is outside the Yh range, or it can be at any position at set point w if within Yh range.

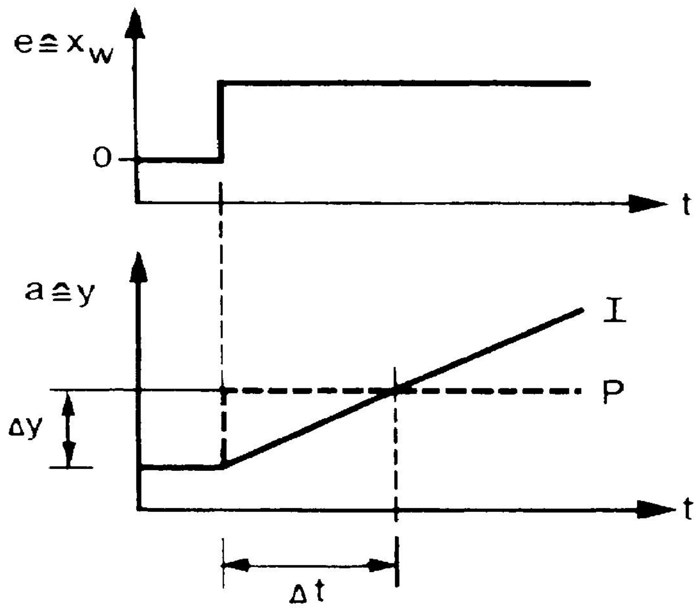


Static characteristic of an I-controller.

Control Theory

Dynamic Behaviour of a I-controller.

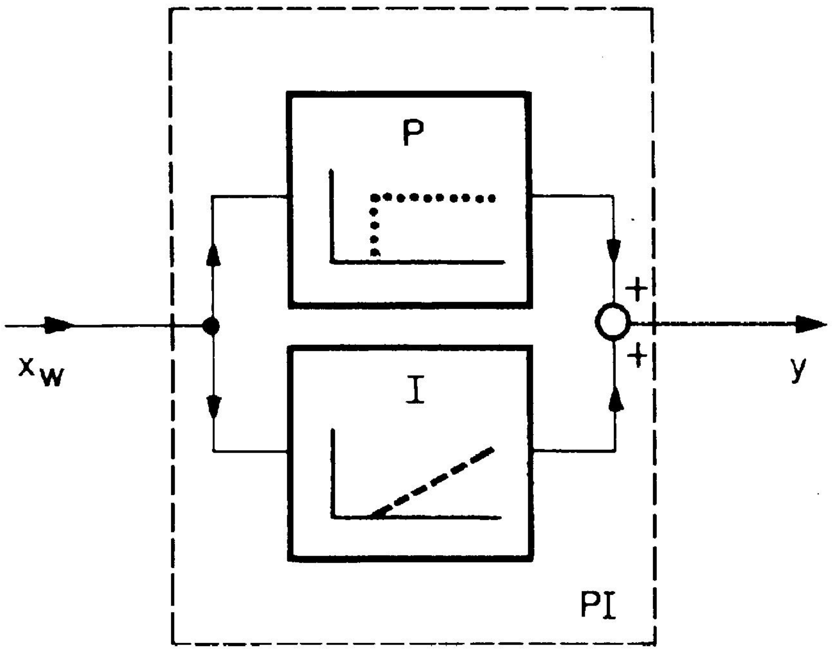
- The following important properties of an I-controller can be derived from the step response function.
- For a step change of the input signal (controlled variable x) the output signal (correcting variable y) starts to change linearly in respect to time until Yh range limit is reached.
- The rate of change of the output value is proportional to the change in the input value i.e. the floating rate r y is proportional to the control deviation Xw.
- Contrary to a P-controller, the control effect is built up slowly , since a certain time r t must elapse until the change in the correcting variable value attains the required r y.
- Thus an I-controller is a slow controller with a long control time, and is very seldom used in HVAC systems.



I Integral controller
P Proportional controller.

Controller with Proportional Integral Action (PI-controller)

- A PI-controller is a combination of a P-controller and an I-controller connected in parallel.
- The advantages of a P-controller (quick reaction) and those of an I-controller (independent of load) are combined together.
- The step function response of a PI-controller can be determined by adding together the step function responses of a P and an I-controller.
- In the case of the P-controller, the P-part causes a change r yp in the correcting value y proportional to the control deviation Xw. (r yp = KR * Xw)
- However, because of the I-part, the valve does not remain in the proportional position, but changes it's position further until the deviation disappears.

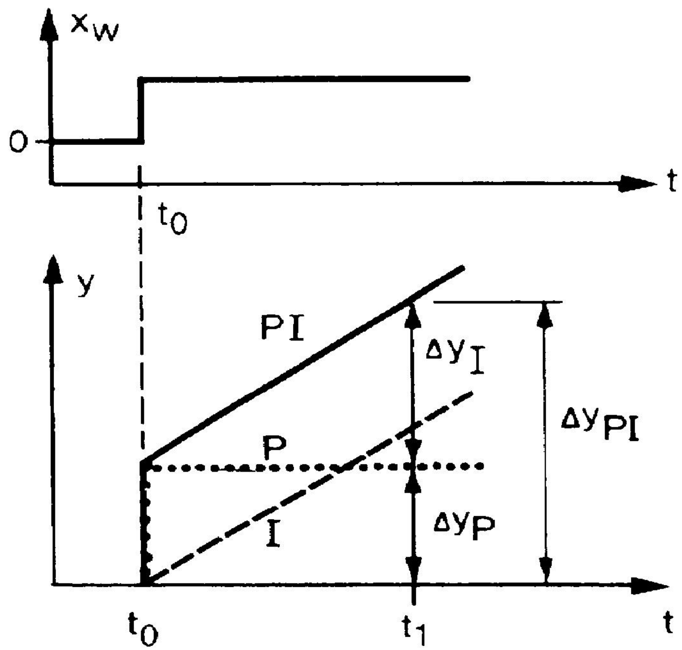


P P-part
I I-part

Control Theory

Controller with Proportional Integral Action (PI-controller)

- The floating rate r y is a function of the integral action factor KI of the I-controller and is proportional to the control deviation xw
  r y = K * xw
- For a given time, (t0…t1) the change r y PI in the value of the correcting variable is given by the sum of the two values:
  r y PI = r yP + r yI = (KR * Xw) + (KI * Xw * t)
- A PI-controller is independent of load because of the I-part, the correcting variable y keeps on changing as long as there is a control deviation Xw.
- As in the case of the I-controller, there is no fixed relationship between the control deviation Xw and the position y of the valve.

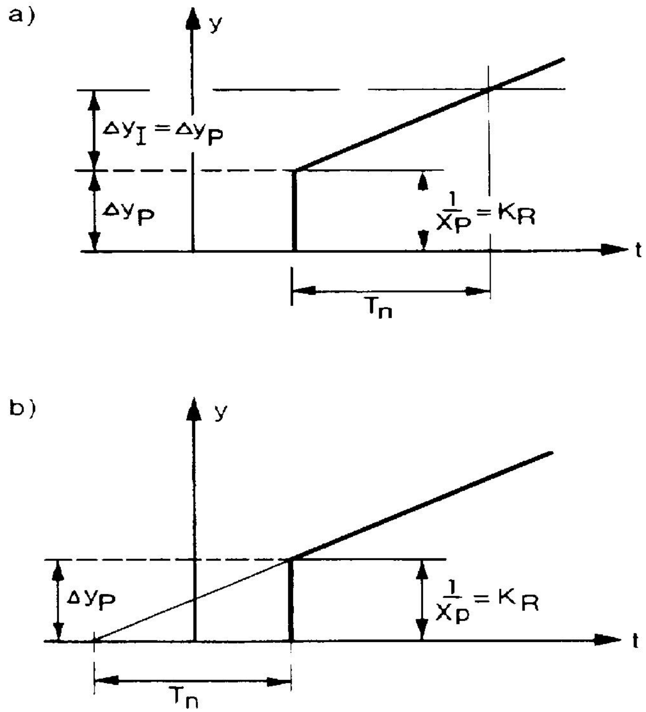


r yP P-part
r yI I-part
r yPI P-part + I-part

Control Theory

Dynamic Behaviour of a PI-controller.

- The dynamic behaviour of a PI-controller is determined by the two characteristic values,
  - The first characteristic value is the throttling range XP, (the proportional control factor KR). It determines the magnitude of the P-part.
  - The second characteristic value is not the integral action factor KI of the I-controller, but the so called integral action time Tn.
- Tn is expressed in seconds or minutes, and can be defined in two ways,
  a) Tn is the time which the I-part needs to bring about the same change in e r yI for the same control deviation Xw as for r yP
  b) The I-controller would need to act earlier by Tn to attain the same change r yP which the P-controller affected immediately.

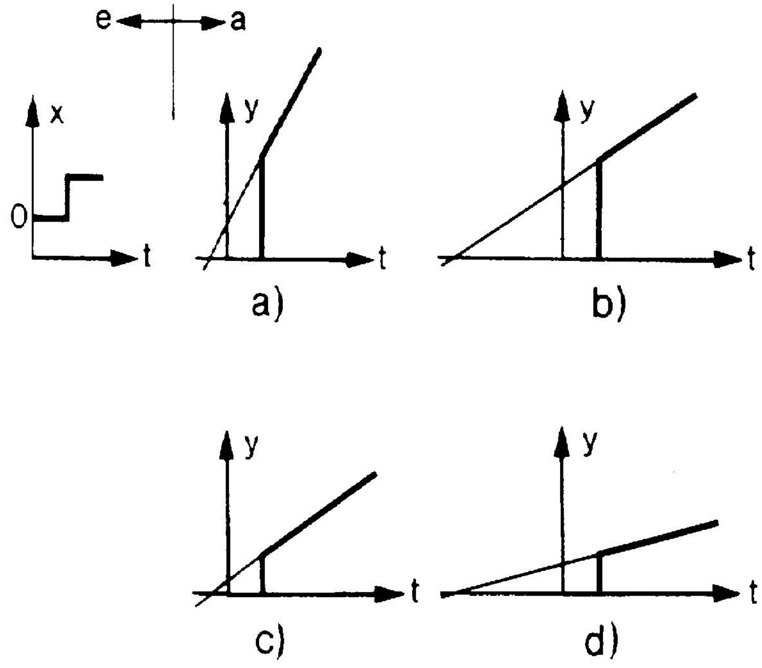


Definition of the integral action time Tn.

Control Theory

Dynamic Behaviour of a PI-controller.

- In most PI-controllers, it is possible to set both the characteristic Xp and Tn within wide ranges to suit various applications.
- This enables adjustment of the dynamic behaviour of the controller to the properties of the controlled system and thus high control quantity.
- For the same input signal, the step function responses for different controller settings are shown.
- Because of the setting possibilities for the throttling range Xp and integral action time Tn, a high quality controller is obtained which can be used for practically all types of applications.
- However, a good knowledge of the theory of control is required to select optimum adjustment for a given system.

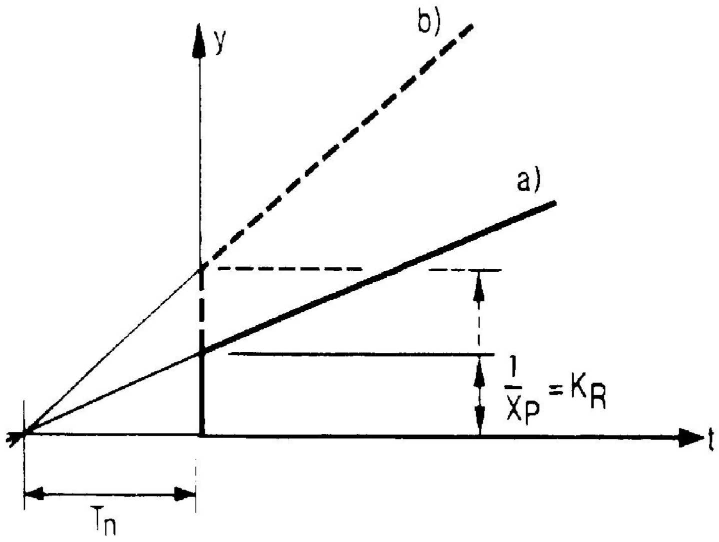


e Input signal
a Output signal
a) Xp small, Tn short
b) Xp small, Tn long
c) Xp big, Tn short
d) Xp big, Tn long

Control Theory

Dynamic Behaviour of a PI-controller.

- In the field of Heating, Ventilating, and Air Conditioning, continuously adjustable integral action time Tn is not always available for a PI-controller.
- Some controllers have wide adjustment of Xp, but only a few selections for integral action time Tn.
- This simplifies the optimum settings, as the Tn selections are divided in to fast and slow plant.
- In most HVAC systems, this method of selection is adequate, as the process is well known and do not vary greatly, as distinct from industrial processes.
- The diagram shows the effect of two Xp settings for the same integral action time Tn.

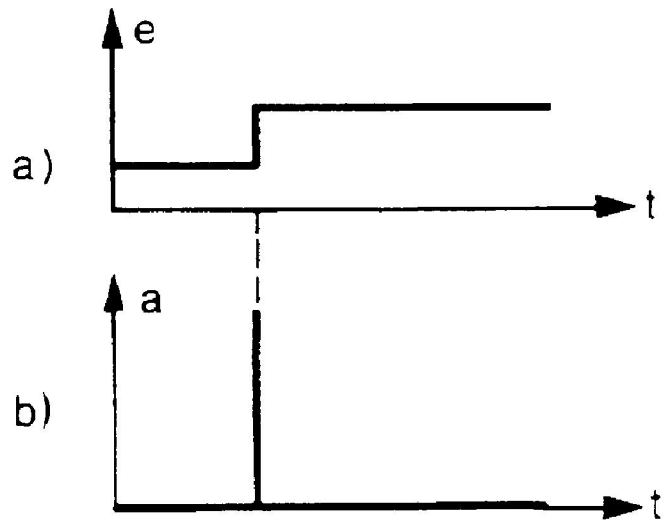


a) Big Xp
b) Small Xp

Control Theory

The Differential Unit (D-unit)

- In a closed loop control system, a quick and correspondingly large change in the value of the controlled variable as a reaction to a control deviation helps to achieve rapid control.
- However, the P, I, and PI- controllers explained so far fulfil this demand only partially.
- This is undesirable in controlled systems with relatively long dead times as it is not possible to reduce the control deviation to zero in a short time.
- To eliminate persistent control deviation, a P or PI-controller is given a "premonition", so at the start of the step change, it reacts more quickly and with more gain than a pure P or PI-controller .
- It is given a differential unit (D-unit) - also known as a derivative unit - that gives the initial "control push".

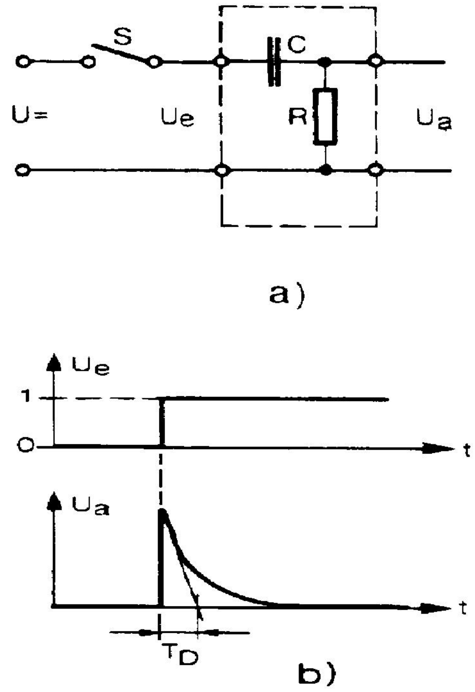


Step function response of an ideal D-unit.
a) Input signal e, step function.
b) Output signal a, theoretical function.

Control Theory

The Differential Unit (D-unit)

- A D-unit is made up of an electronic circuit with with a condenser C and a resistance R .
- The direct voltage Ue on the left side is the input value and the voltage Ua measured across resistance R is the output value.
- When switch S is closed, there is a quick steep change in the output value Ua, and thereafter drop with the differential time constant TD exponentially.
- This example explains the D-unit action, but the CR time constant method is still used in hardware based controllers and equivalent outputs are also developed in the software of software based controllers.

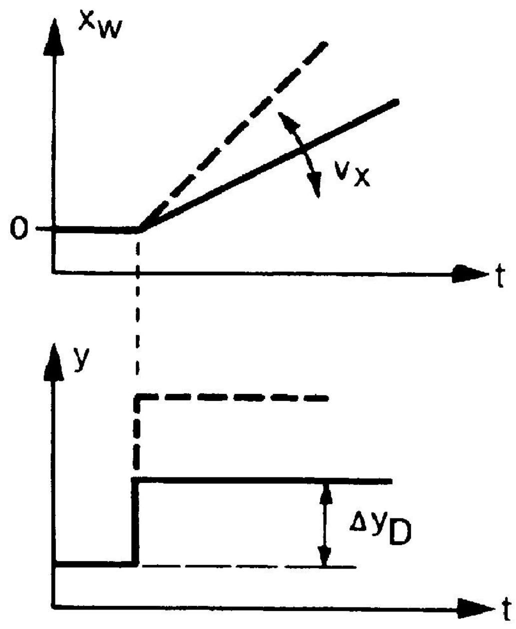


a) Electronic circuit.
b) Step function response.

Control Theory

Ramp Function Response of a D-unit.

- At the beginning of a change in controlled value X, the output value r yD is adjusted to the D-unit setting.
- While the controlled value X continues to change at constant rate, the r yD value remains constant.
- A further increased change in controlled value X will adjust r yD to a proportional increased value.
- In the steady state, the value of the control variable is not measured by a D-unit, therefore this alone cannot be used for control purposes.
- However, as an auxiliary device to P and PI-controllers, it plays an important role in the control of plants with long dead times.
- The D-unit gives the controller "derivative action" i.e. it reacts as if it had detected the change in the value of the controlled variable earlier. The harmful effects of dead time are diminished.



Vx Rate of change of the controlled variable

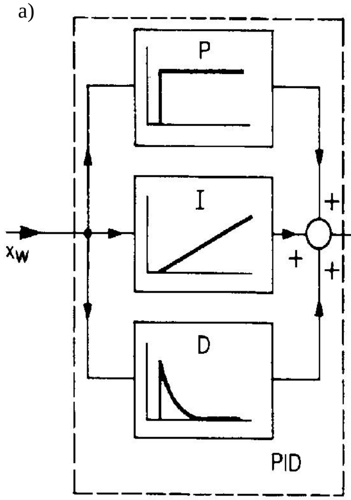
r yD Change in the value of the correcting variable in relation to he ramp control.

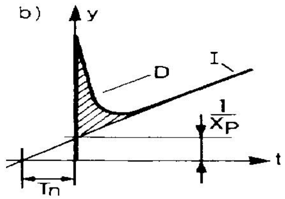
Control Theory

Controller with Proportional Integral Differential Action (PID-controller)

- A PID-controller is a combination of a P-controller, an I-controller, and a D-unit.
- For the formation of the output signal of the controller, not only is the magnitude of the control deviation taken into account, but also it's rate of change by the D-unit.
- The step function response of a PID-controller consists of an added signals of the P-part, I-part, and D-part.
- At the beginning, the D-unit causes a big change in the value of the correcting variable. This avoids the formation of a too large a control deviation following a disturbance.
- Once the initial rate of change disappears, the D-unit value falls to zero, the correcting value Y falls and meets the rising I-part output, and continues along the I - part output until the deviation is eliminated.

a)



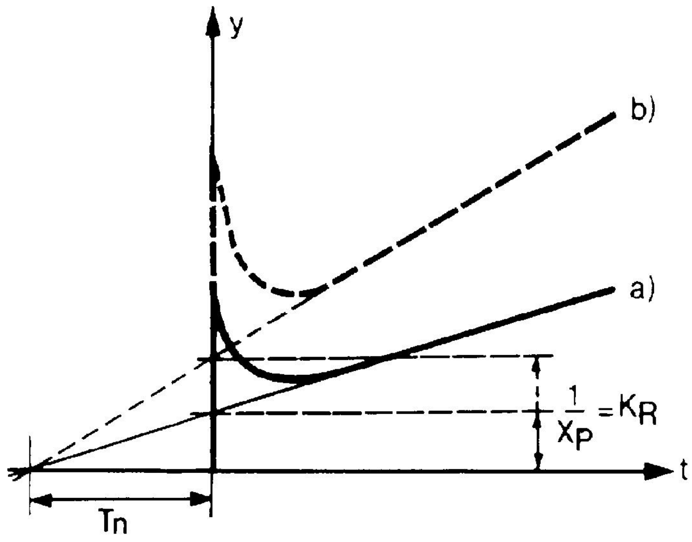


a) Model b) Step function response
P P-part I I-part D D-part

Control Theory

Dynamic Behaviour of a PID-controller.

- The dynamic behaviour of a PID-controller is a determined by 3 characteristic values.
- The proportional control factor Kr of the P-part (Xp)
- The integral action time Tn of the I-part.
- The derivative action time Tv of the D-part.
- HVAC systems fall into predictable plant functions, and in many PID-controllers only the XP function is adjusted, with the integral action time Tn and the derivative action time remain constant.
- This is the case in the old POLYGYR, but individual adjustment is available in the POLYGYR Joker .
- A good knowledge of control theory is required to tune in] complex plant, and you will find examples of adjustments in the POLYGYR joker documentation.

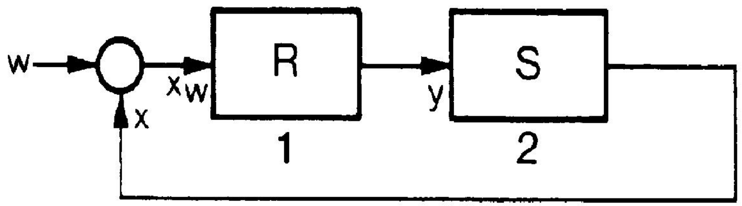


Step function response of a PID-controller with fixed Tn and Tv but variable Xp.
a) Big Xp
b) Small Xp

Control Theory

Control Loop.

- Up to now, the controlled systems and the controllers have been dealt in detail separately .
- A control loop is formed by combining a controlled system and a controller.
- The block diagram shows the influences around the loop, and flow is in one direction only.
- Since the controlled variable and the correcting variable influence each other mutually, a closed loop is a system prone to oscillations.
- Three types of oscillations found in systems are:
- Damped oscillations - amplitude decreases.
- Undamped oscillations - amplitude remains constant.
- Excited oscillations - amplitude increases.



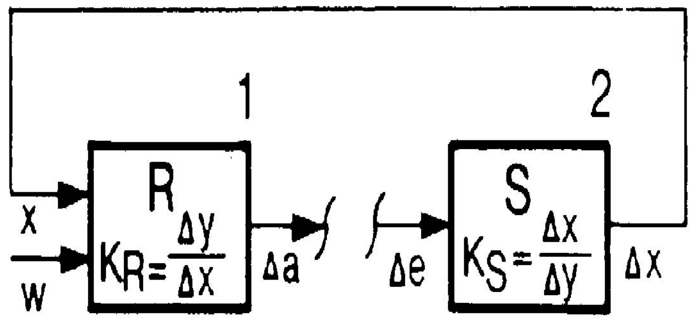
Block diagram of a control loop.
1. Controller
2. Controlled system.

W Reference value.
X Controlled variable.
Xw Control deviation.
y Correcting variable.

Control Theory

Control Loop Gain Vo.

- Oscillatory behaviour in a control loop with a P-controller and a controlled system is a function of the degree of difficulty and the closed loop gain V0.
- The closed loop gain V0 of a control loop is given by the product of the proportional control factor KR of the controller and the proportional control control factor KS of the controlled system.
- To explain closed loop gain , the control loop is cut between the P-controller and the controlled system.
- Step change of r e (input to valve) produces room change r x , and via controller r a output to r e input.
- If the controller output r a has too much gain, it will open the valve too far for a given room change, and create undamped or excited oscillations.
- The desired closed loop gain is set by changing the proportional control factor KR = 1 / Xp.



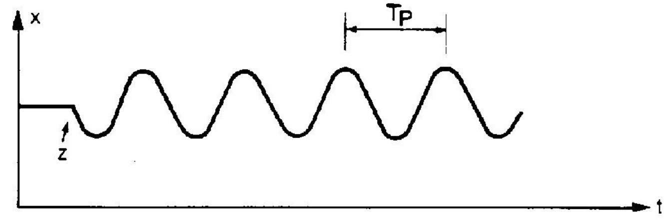
Definition of the closed loop gain V0.
1. Controller
2. Controlled system.

r e Change in the input value, correcting variable.
r a Change in the output value, correcting variable.
r x Change in the value of the controlled variable.
r y Change in the value of the correcting variable.
KR Proportional control factor of the P-controller
KS Proportional control factor of the controlled system.
V0 Closed loop gain.

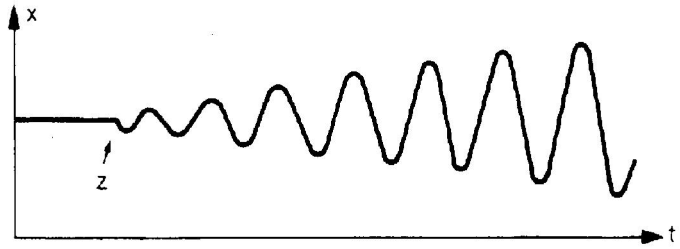
Control Theory

Transient Response.

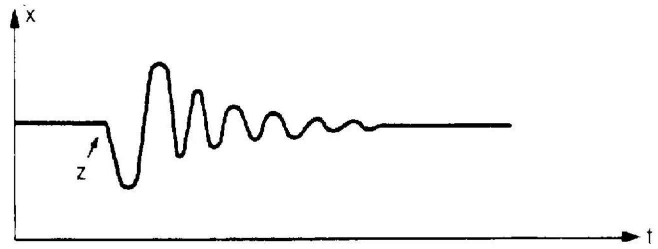
- In a control loop without any external influence, if the controlled variable does not assume a constant value even after a long time, this is self- sustained oscillation. Correct setting of Xp avoids this action.
- Undamped oscillations have constant amplitude and frequency with continuous oscillations following **z** disturbance.
- Time required for a full oscillation is known as the oscillation period Tp. The closed loop gain of undamped oscillations is called V0 critical.
- The excited oscillations are characterised by the increasingly growing amplitude of the continuous oscillation. This happens when V0 > V0 critical.
- The damped oscillations are characterised by the decreasing amplitude until zero. Steady state is achieved when V0 < V0 critical.



Undamped oscillation.
TP Oscillation period. Z Disturbance.



Excited oscillation.

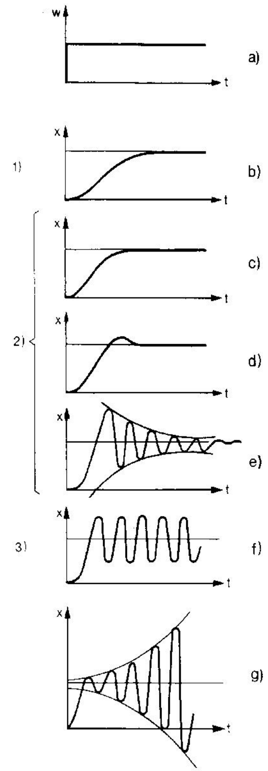


Damped oscillation.

Control Theory

Transient Response.

- Control loop tends to oscillate, if the controller and hence the closed loop gain, is incorrectly tuned to the controlled system.
- In the case of controlled systems of higher order and dead time , the closed loop gain cannot be increased arbitrarily since otherwise the control process will be damped insufficiently or can be undamped.
- On the other hand, too low values of the closed loop gain make the control loop inert and inaccurate.
- In order to obtain a sufficient damping of the control process, a simple, commonly used rule recommends that the closed loop gain V0 be made half as big as the critical closed loop gain.

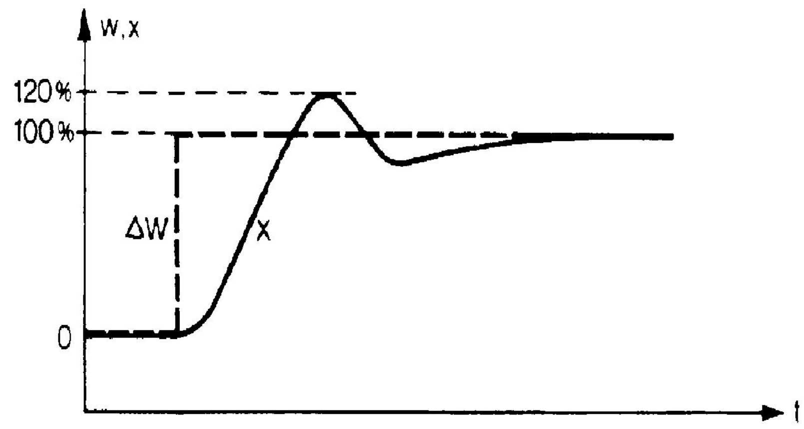


Different transient response.
a) Set value step b) Over-critically damped
c) Critically damped d) Sub-critically damped
e) Periodically damped f) Periodically undamped.
g) Periodically excited.
1. Too inert 2. Satisfactory to good.
3. Stability limit.

Control Theory

Dampening Behaviour.

- The function of the control device is to take corrective action for the changes in set value and disturbances, as quickly as possible.
- Undesirable oscillations must be avoided. Some plant can accept some oscillation, i.e. supply air temperature control, but excessive oscillations in a boiler could cause the safety thermostat to react.
- Experience has proved that the most suitable setting values for the controller (shortest possible control time) given by Ziegler and Nichols are those that after a step change the controlled variable over-swings by about 20% and does not change direction more than time.
- This represents a sub-critically damped ,periodic transient response. Fast and without excessive over-swing.



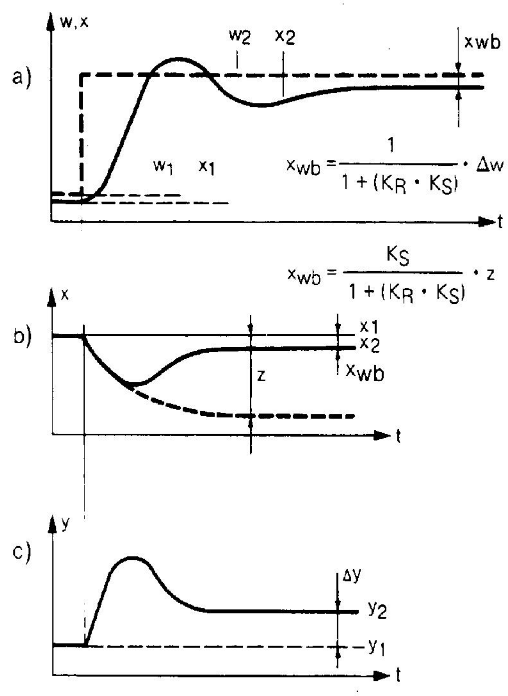
Optimum transient response according to Ziegler and Nichols.

r W Change in the reference value.
X Controlled variable.

Control Theory

P-controller and the Controlled System.

- A P-controller is not able to eliminate completely the deviations caused by the changes in the set value and the disturbances.
- There will always be the so-called steady state deviation (offset), the magnitude of which is a function of the corresponding change in the disturbance, the selected throttling range Xp and the position of the working point.
- A small throttling range means a small steady state deviation, but on the other hand, the control loop tends to oscillate.
- A large throttling range results in high stability, but also in a high inertia and load dependence i.e. a large steady state deviation .
- A P-controller should not be used in fast acing and difficult controlled systems due to large XP required.

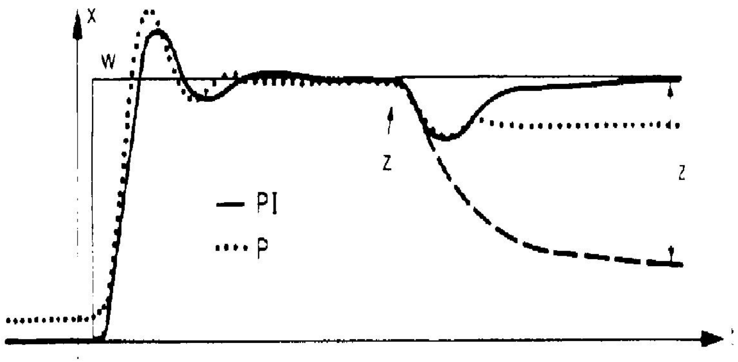


Control processes with a P-controller.
a) Change in set value.
b) Disturbance behaviour Z.
c) Correcting value change due to disturbance Z.
w1,w2 Initial and subsequent set values.
x1, x2 Initial and subsequent value of controlled variable.
Xwb Steady state P-deviation (offset)
y1,y2 Initial and subsequent values of the correcting values.

Control Theory

PI-controller and the Controlled System.

- A PI-controller combines the advantages of a P-controller (quick acting) and an I-controller (independent of load).
- Initially a control deviation is eliminated quickly, but only roughly, by the P-part. The integral part effects the control process over time until the set value(set-point) is attained.
- The PI-controller is adjusted to the controlled system by adjusting the throttling range and integral action time Tn
- The larger the Xp setting, the control process becomes too slow, as it takes too long for I-part to eliminate offset.
- The higher the integral action time Tn , the offset will remain too long to eliminate the offset.
- A PI-controller should be set to the correct Xp, to the highest possible Tn and adjust Tn as necessary

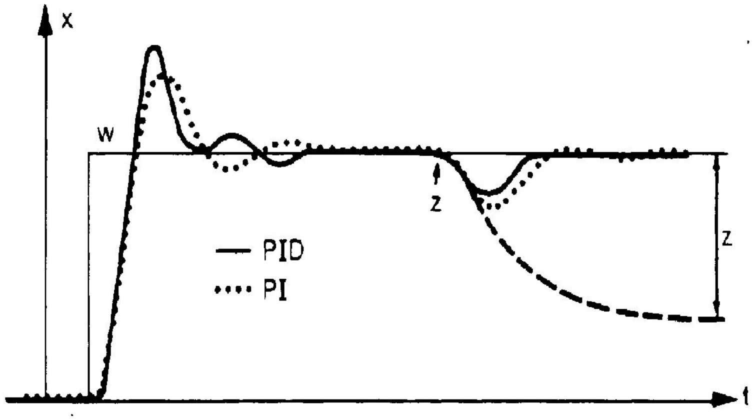


Step change in the set value and a disturbance z controlled by a P and PI-controller
PI PI-controller
P P-controller
z Disturbance

Control Theory

PID-controller and the Controlled System.

- PID-controllers are used mostly for systems which have a high degree of difficulty (relatively long delay time) and which the controller must eliminate the deviations quickly.
- The control deviation is measured by the P-part, the rate of change of the deviation is measured by D-part, and the offset is eliminated by the I-part.
- The PID-controller can be difficult to adjust as the P, I, and D settings must have the correct relationships to suit the controlled system.
- P and I setting requirements are similar to a PI-controller.
- If the derivative action time Tv is short, the D-unit cannot develop it’s desired effect.
- If the derivative action time Tv is long, it leads to astable and ineffective control behaviour.

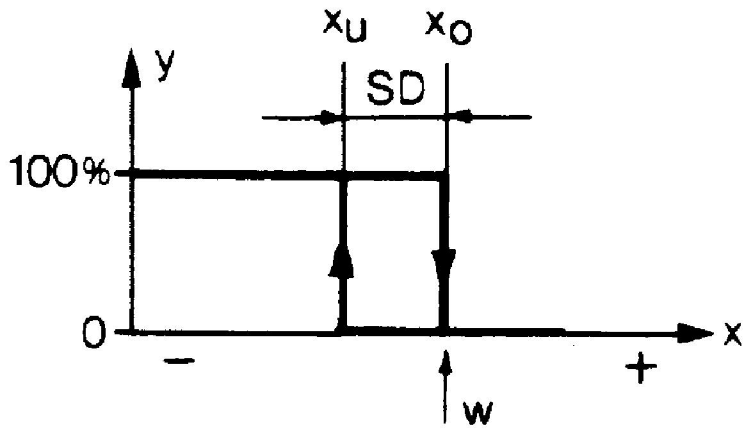


Step change in the set value and a disturbance z controlled by a PI and PID-controller
PID PID-controller
PI PI-controller
z Disturbance

Control Theory

On / Off (Two Position) Controller.

- The static characteristic of an on / off controller shows the relationship between the controlled variable x and the correcting variable y .
- The correcting variable y can take up only 2 fixed values i.e. y = 100% (on) and y = 0% (off).
- The SD switching differential is the difference in x value between the on and off switching points. Xu is the upper and xo is the lower switching point.
- W set-point is usually set at the off position of the switching differential.
- Calibration of SD differential, and w set-point is available on most on / off controllers.

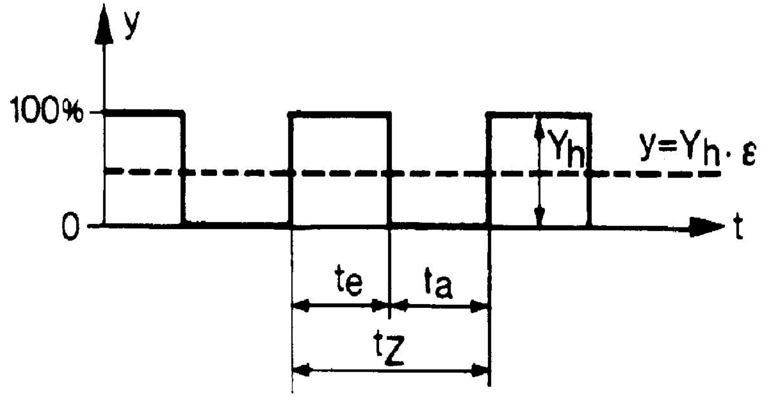


Static behaviour of an on/off controller
x Controlled variable
xu Lower switching point
xo Upper switching point
y Correcting variable
w Set value
SD Switching variable.

Control Theory

On / Off (Two Position) Controller.

- The working principle of the on/off controller is that the controlled plant receives the full output, and the duration of the on period is a function of the load.
- The greater the load the longer will the be the duration of the on period.
- The quotient resulting from the duration te of the ON period and the cycle time tZ, and is known as the switching ratio ε.
  Switching ratio $\varepsilon = \frac{te}{tZ}$
- The theoretical value y of the correcting variable is the dotted line resulting from the switching cycle.
- The ON/OFF controller is generally used as an inexpensive and safety control for simple plant.



Switching cycle of an on/off controller.
te On duration, pulse duration
ta Off duration
tz Cycle time.
ε Switching ratio.

Control Theory

7. Alphabetic summary of the general terms

Actuator:	It changes the position of the correcting unit in the appropriate direction in function of the output signal of the controller (e.g. electric motor).
Compensating time T_g:	Characteristic of a controlled system of higher order.
Control, to:	Process in which the value of the variable to be controlled (the controlled variable) is measured and compared with the reference value continuously and — depending upon the result of this comparison — is changed in the direction of the reference value. The process takes place in a closed loop, the control loop.
Control characteristic:	Graphic illustration of the relationship between the controlled variable x and the correcting variable y in steady state.
Control deviation x_W:	Difference between the controlled variable x and the reference value w (set value) expressed in the units of the controlled variable $x - w = x_W$.
Controlled system S:	The plant to be controlled, i.e. the plant in which the controller has to maintain the controlled variable x at a constant value in spite of the undesirable disturbances: — Input value: Correcting variable y — Output value: Controlled variable x.
Controlled variable x:	The physical variable which must be maintained at a given value in a controlled system, i.e. which is controlled (temperature, humidity etc.). It is the output value of the controlled system and the input value for the controller.

Control Theory

Controller R:	The device which initiates the control process in the controlled system, i.e. which determines the difference between the measured and set values of the controlled variable and operates the correcting unit correspondingly to eliminate the deviation. – Input value: Controlled variable x – Output value: Correcting variable y.
Control loop:	Combination of the controlled system and the controller acting uni-directionally.
Control process:	Its purpose is to bring a physical variable (the controlled variable x) to a pre--determined value (reference value w) and maintain it at this value in spite of all disturbances.
Control range X_h of the controlled variable:	Change in the value of the controlled variable, if the correcting variable changes from 0 ... 100 % (in units of the controlled variable).
Correcting range Y_h of the correcting variable:	Range of the maximum possible change in the value of the correcting variable: – General: 0 ... 1 or 0 ... 100 % – Specific: e.g. 0 ... 20 mm.
Correcting unit:	A unit incorporated in the control loop to administer the required amount of energy or volume (e.g. valve).
Correcting variable y:	A variable, the value of which can be changed by the controller (e.g. the valve travel) and which effects the value of the controlled variable. It is also the output value of the controller and input value to the controlled system.
Dead time T_t:	Characteristic of a controlled system of the first order (transport time).
Degree of difficulty λ:	Relationship between the dead time T_t and the time constant T_S, or between the delay time T_u and the compensating time T_g: $\lambda = T_t/T_S$ or T_u/T_g.

Control Theory

Delay time T_u: A characteristic of controlled systems of higher order.

Detector: A unit which measures the value of the controlled variable.

Disturbance value z: An external variable which influences the control loop (e.g. extraneous heat, solar radiation etc).

Dynamic behaviour: Relationship between the changes in the values of the output and input signals as a function of time.

Measured value: The momentary value of the controlled variable measured by the detector.

Open loop control: One or more input values influence other variables as output values in an open loop.

Proportional control factor K: Relationship between the changes in the values of the output and input signals:

$K = \Delta a/\Delta e$

- For controller: $K_R = \Delta y/\Delta x$
- For the controlled system: $K_S = \Delta x/\Delta y$

Reference value range W_h: Generally corresponds to the scale range of the setting unit.

Reference value w: Variable which determines the momentary set value. One differentiates between:

- Fixed value control: Reference value is constant and identical with the set value.
- Programmed control: Set value is dependent upon the time of the day.
- Slave control: Set value of the control process is determined by another measured variable.

Set value: The momentary desired value of the controlled variable which must be maintained constant in spite of the varying disturbances, or the value set on the setting unit.

Static behaviour: Relationship between the changes in the values of the output and input signals in the steady state.

Step function: A sudden change in the input value by an arbitrary amount.

Step function response: Time behaviour of the output signal resulting from a step function at the input.

Time behaviour: It indicates the manner in which an output signal follows a varying input signal as a function of time.

Time constant T_S: A characteristic of a controlled system of the first order.

Transient behaviour: It describes the time behaviour of the output signal for the characteristic time behaviour of the input signal.

Transient response: Time behaviour of the output signal following a step function of the input (= step function response).

Control Theory

8. Summary of the signs used in the formulae

a	Output signal
e	Input signal
K_D	Differential action factor
K_I	Integral action factor
K_R	Proportional control factor of a P-controller
K_S	Proportional control factor of a controlled system
Q	Load
R	Control factor
SD	Switching differential
T	Time constant (general)
t	Time (current)
t_o	Initial time
T_D	Differential time
T_g	Compensating time
T_I	Integral time
T_n	Integral action time
T_P	Oscillation period
T_S	Time constant of the controlled system
T_t	Dead time
T_u	Delay time
T_v	Derivative action time
T_y	Corrective time
$\dot{V}$	Rate of flow
V_o	Closed loop gain
v_x	Rate of change of the controlled variable
v_y	Correcting rate
w	Reference value
W_h	Reference value range, setting range
x	Controlled variable
x_d	Control differential ($= w - x$)
X_h	Control range of the controlled variable (control range)
X_P	Throttling range
x_w	Control deviation ($= x - w$)
x_{wb}	Steady state P-deviation
y	Correcting variable
Y_h	Correcting range of the correcting variable
z	Disturbance value
Δ	Difference
ϵ	Switching ratio
λ	Degree of difficulty

Control Theory

References

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